NEWTON-GMRES AND -MINPERT BEHAVE ASYMPTOTICALLY ALMOST THE SAME

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Abstract

The MINPERT algorithm [2] is a Krylov method for solving large linear systems $Ax = b$; it is based on backward error minimization properties, since it determines an approximation x_m^{MP} by

$$\min_{x_m \in x_0 + \mathcal{K}_m} \|[\Delta_A \ \Delta_b]\|_F \quad \text{subject to} \quad (A - \Delta_A)\, x_m = b + \Delta_b.$$

The GMRES method [3] can also be viewed as being based on backward error minimization:

$$\left\|b - Ax_m^{GM}\right\|_2 = \min_{x_m \in x_0 + \mathcal{K}_m} \|b - Ax_m\|_2 = \min_{x_m \in x_0 + \mathcal{K}_m} \{\|\Delta_b\|_2 : Ax_m = b - \Delta_b\}.$$

We present some results relating the behavior of these two methods when used to solve the linear systems from the Newton method.

More exactly, assuming that the Newton-GMRES iterates converge to a solution of a nonlinear system, solving the same linear systems but with MINPERT (using the same parameters as for GMRES) one obtains that under certain conditions the difference between the normalized corrections tends to zero; the same result holds inverting the roles of GMRES and MINPERT [1].

In the same setting we show that, only under the existence and convergence conditions, the quotient of the norms of the residuals corresponding to these corrections tend to 1.

References

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