ON FAST REGISTRATION SCHEMES WITH APPLICATION TO MEDICAL IMAGING

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Keywords: Image registration, Euler-Lagrange equation, direct methods

Abstract

Image registration is one of the most challenging task within digital imaging, in particular in medical imaging. In most applications simple rigid deformations are not satisfactory and complex, non-rigid and non-linear deformations must be employed.

Given a *template image* T and a *study image* S, where $T, S : \Omega = [0,1]^d \to R$, the purpose of the registration is to determine a transformation, sometimes called warping, of T onto S. Ideally, one wants to determine a *displacement field* $u : R^d \to R^d$ such that $T(x - u(x)) = S(x)$. The question is how to find such a mapping u. A commonly used approach is to minimize the following functional

$$\mathcal{J}[u] := \frac{1}{2} \int_\Omega \left(T(x - u(x)) - S(x)\right)^2 dx + \mathcal{S}[u]$$

where $\mathcal{S}$ denotes a suitable *smoothing* or *regularizing* term.

To compute a minimizer of the functional $\mathcal{J}$ we apply the calculus of variations and obtain a non-linear partial differential equation: the associated *Euler-Lagrange equation*. Subsequently we solve this equation by a finite difference approximation accompanied by a time-marching or fixpoint scheme. It should come as no surprise, that the main work in the overall scheme is the repeated solution of a (highly structured) linear system.

In this talk we consider various choices for the smoother $\mathcal{S}$ and show how to solve the resulting linear system fast and robust by means of direct schemes.

Furthermore we demonstrate the performance of the presented approaches for a variety of academic and real life examples. These examples include the registration of histological sections of a human brain, of 3D MR-mammography images of a female breast, and of FMR-images of a human visual cortex, respectively.