BLOCK AND BAND LANCZOS SOLVERS: A REVIEW OF OPTIONS

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Abstract

Since the early work of Cullum and Donath (1974) and Underwood and Golub (1975,1977), there has been, from time to time, a renewed interest in conjugate gradient methods for systems with multiple-right hand sides and in Lanzos methods with several starting vectors for solving linear systems, computing eigenvalues, or, more recently, system identification and model reduction.

While the formal definition of block versions of CG and Lanczos is quite easy, the true difficulties come when a dimension reduction of the block Krylov space (so-called deflation) has to be treated properly, or, in the nonsymmetric case, when the block Lanczos process breaks down. Deflation for symmetric systems was treated by Nikishin and Yeremin (1995) (and, to some extent, by others before). For the nonsymmetric case, the answer to both difficulties lies in an adjustment of the look-ahead Lanczos algorithm for single systems. The resulting nonsymmetric look-ahead (or cluster) band Lanczos algorithm has been developped in 1994–1996 in partly independent and partly cooperative efforts by Aliaga, Boley, Feldman, Freund, Hernández, Malhorta, and even others. The work culminates in the paper by Aliaga, Boley, Freund, and Hernández in *Math. Comp.* (2000, submitted 1996).

One aspect of block solvers and look-ahead procedures alike is that there is a trade-off between cost and stability. The purpose of this talk is to discuss this issue and to raise questions about various implementation options.