SUBSPACE PRECONDITIONING FOR DISCRETE ILL-POSED PROBLEMS

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Abstract

We present a two-level iterative method, based on conjugate gradients, for the solution of large-scale discrete ill-posed problems. The algorithm is algebraically equivalent to the two-level Schur CG algorithm of Hanke and Vogel [1] but involves less work per iteration. The key idea in the algorithm is to project the problem onto a low-dimensional subspace, spanned by "smooth" basis vectors, which contains a dominating component of the regularized solution. The remaining component is then computed via the Schur complement.

We first review the algorithm and demonstrate how it should be implemented efficiently and reliably, and then we demonstrate its usefulness with numerical examples that give insight into the proper use of the algorithm. Finally we discuss the choice of stopping criteria for the iterations.

References

[1] M. Hanke and C. R. Vogel, *Two-level preconditioners for regularized inverse problems I: Theory*, Numer. Math., 83 (1999), pp. 385–402.

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