PRECONDITIONING TECHNIQUES FOR NEWTON'S METHOD FOR THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

Howard Elman
Department of Computer Science,
University of Maryland
College Park, MD 20742 USA ,
e-mail: elman@cs.umd.edu

Daniel Loghin
Oxford University Computing Laboratory
Parks Road, Oxford, OX1 3QD
e-mail: daniel.loghin@comlab.ox.ac.uk

Andy Wathen
Oxford University Computing Laboratory
Parks Road, Oxford, OX1 3QD
e-mail: wathen@comlab.ox.ac.uk

Keywords: Block preconditioners, Navier-Stokes, Newton's method.

Abstract

Newton's method for the incompressible Navier-Stokes equations in $\mathbb{R}^{\mathrm{d}}$ gives rise to large sparse non-symmetric indefinite matrices with the general block form

$$\begin{pmatrix} F & B_1^T \\ B_2 & C \end{pmatrix} \tag{1}$$

where F has $(d \times d)$ blocks and is non-symmetric and in general indefinite. In this work we investigate the performance of two preconditioning techniques introduced for the Picard method for which both proved significantly superior to other approaches such as the Uzawa method. The first, introduced in [1], is a block preconditioner which is based on the algebraic structure of (1). The other approach, described in [2], uses also a block preconditioner which is derived by considering the underlying partial differential operator matrix. Some analysis and numerical comparison of the methods will be presented.

References

[1] H. C. Elman, Preconditioning for the Steady-State Navier-Stokes Equations with Low Viscosity, *SIAM J. Sci. Comp.*, 20:1299–1316, 1999.

[2] D. Kay, D. Loghin and A. J. Wathen, A Preconditioner for the Steady-State Navier-Stokes Equations, to appear in *SIAM J. Sci. Comp.*, 2002.

Acknowledgement: The research of the second and third authors was supported by a joint EPSRC–British Energy plc. grant.