A NULL-SPACE METHOD APPROACH IN SOLVING DARCY'S EQUATIONS

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Keywords: augmented systems, sparse matrices, mixed finite-element, graph theory

Abstract

We present a Null Space method which uses a mixture of direct and iterative solvers applied to the solution of the augmented system produced by a Mixed Finite-Element approximation of the Darcy's Law [1].

The direct solver is based on the LU factorization of the submatrix of the augmented system which approximates the divergence operator div . The basic structures of the matrices involved are described in terms of graph theory and any spanning tree of the graph underlying the divergence matrix identifies row and column permutations by which the required LU decomposition is directly obtained. Furthermore, the spanning tree allows us to describe the block structure of the projected Hessian matrix on which we apply the conjugate gradient algorithm.

We compare the "Shortest Path Tree" algorithm used in [1] with other techniques for computing the spanning tree, such as the "Minimum Cost Spanning Tree" algorithm. The aim is to identify the fastest way to achieve the identification of a spanning tree by which we can compute efficient preconditioners.

We point out that our null space method is an algebraic approach to the computation of the finite-element approximation of $H(\text{curl}\,)$ which characterizes the subspace of the divergence-free vector fields in $H(\text{div}\,)$. Nevertheless, this method does not require the explicit memorization of the null space and, therefore, of the related finite-element approximation of $H(\text{curl}\,)$.

We compare the performance of our prototype version of the algorithm with a well established direct solver and we conclude that even if our implementation can be ten times slower than the direct solver, the absence of fill-in makes our code competitive for large problems and promising for the 3-D case.

References

[1] M. Arioli and G. Manzini. A network programming approach in solving Darcy's equations by mixed finite-element methods, Tech. Rep. IAN-1253, IAN, 2001.