PRECONDITIONED ITERATIVE METHODS FOR IMAGE RESTORATION

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Abstract

Image restoration is an example of an "ill-posed problem", which can be defined as a problem that does not have a unique solution, or the solution is not a continuous function of the data. Such problems are extremely sensitive to perturbations (noise) in the data; that is, small perturbation of the data can lead to arbitrarily large perturbations in the solution. Many algorithms have been developed to compute approximate solutions of ill-posed problems, but they may differ in a variety of ways. For example, there are several different regularization methods one could use, and for each of these, various different methods for choosing a regularization parameter. For structured, large-scale problems, an attractive approach is to use certain iterative methods, such as the conjugate gradient algorithm, where regularization is enforced through early termination of the iterations. In some cases, preconditioning can be used to accelerate convergence. However, if not done carefully, preconditioning can lead to erratic convergence behavior and result in fast convergence to a poor approximate solution. In addition to discussing aspects of preconditioning conjugate gradients for ill-posed problems, an alternative scheme based on steepest descent is considered. It will be demonstrated that, with preconditioning, the steepest descent approach is competitive in terms of convergence rate with preconditioned conjugate gradients, with a much less erratic convergence behavior. Moreover, the steepest descent approach has the additional advantage of enforcing nonnegativity, which may be important in some image restoration applications.