CONVERGENCE OF INEXACT RAYLEIGH QUOTIENT ITERATIONS AND JACOBI-DAVIDSON

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Abstract

We consider the computation of the smallest eigenvalue and associated eigenvector of a an Hermitian positive definite pencil.

Rayleigh quotient iterations (RQI) are known to converge cubically and we first analyze how this convergence is affected when the arising linear systems are solved only approximately. We obtain an upper bound on the convergence factor which we show to be sharp for a wide set of error vectors, indicating that the analysis takes properly the errors into account.

We next consider the Jacobi-Davidson method. It acts as an inexact RQI method in which the use of iterative solvers is made easier because the arising linear systems involve a projected matrix that is better conditioned than the shifted matrix arising in classical RQI. We show that our general convergence result straightforwardly applies in this context and permits to trace the convergence of the eigenpair in function of the number of inner iterations performed at each step. We also establish a relation between the residual norm reduction during the course of inner iterations and the norm of the residual associated to the next approximate eigenpair. From a practical point of view, this allows to set up a proper stopping strategy, exiting precisely at the moment where further progress would be useless with respect to the convergence of the outer process.

References

[1] Y. Notay, *Combination of Jacobi-Davidson and conjugate gradients for the partial symmetric eigenproblem*, Numer. Lin. Alg. Appl., 9 (2002), pp. 21–44.

[2] Y. Notay, *Convergence analysis of inexact Rayleigh quotient iterations*, Tech. Rep. GANMN 01–02, Université Libre de Bruxelles, Brussels, Belgium, 2001. *http://homepages.ulb.ac.be/~ynotay/*.

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