PARTIAL ORDERING AND CONVERGENCE CONDITIONS FOR ITERATIVE METHODS BASED ON SPLITTINGS

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Abstract

In the last decades, the study of convergence conditions for the iterative methods based on splittings to solve de linear system $Ax = b$, has arisen in the works of many authors. We can consider that there are two principal kind of matrices: the nonnegative matrices, studied by authors as Varga [7], Berman and Plemmons [1], Marek and Szyld [5] and more recently by Climent and Perea [3], and the symmetric positive definite matrices studied by different authors (see for example, Ortega [6], and more recently Climent and Perea [4]).

To establish the convergence of an iterative method, it is customary to use the partial ordering induced by the corresponding kind of matrices: $A \leq B$, that is, $B - A \geq 0$ for nonnegative matrices, and $A \preceq B$, that is, $B - A$ is a symmetric positive definite matrix for such matrices.

If we compare the convergence conditions established for the above kind of matrices (see Climent and Perea [3,4]), it is easy to see that such conditions are the same, although the proofs are different depending on the kind of matrices considered.

In this work we introduce the necessary additional properties for an arbitrary partial ordering that allow us to establish the convergence conditions for the iterative method $x^{(k+1)} = M^{-1}Nx^{(k)} + M^{-1}b$ where $A = M - N$ is a splitting of A. Also, we establish different partial ordering that satisfy such properties.

References

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