FAST SUMMATION AT NONEQUISPACED KNOTS BY NFFTS

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Abstract

The fast computation of special structured discrete sums or from the linear algebra point of view of products of vectors with special structured dense matrices is a frequently appearing task in the study of particle models, in the numerical solution of integral equations (or of partial differential equations by recasting them as integral equations) and in the approximation of functions by radial basis functions. We develop a new algorithm for the fast computation of discrete sums

$$f(y_j) := \sum_{k=1}^{N} \alpha_k K(y_j - x_k) \quad (j = 1, \ldots, M)$$

based on the recently developed fast Fourier transform at nonequispaced knots (NFFT). Our algorithm, in particular our regularization procedure, is simply structured and can easily be adapted to different kernels K, e.g.

$$\frac{1}{x},\ \frac{1}{|x|},\ \frac{1}{x^2},\ x^2 \log |x|$$

Our method utilizes the widely known FFT and can consequently incorporate advanced FFT implementations. In summary it requires $\mathcal{O}(N \log N + (N+M))$ arithmetic operations. We prove error estimates to obtain clues about the choice of the involved parameters and present numerical examples in one and two dimensions.