INEXACT BLOCK PRECONDITIONERS FOR SADDLE-POINT PROBLEMS

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Abstract

Several numerical application problems require the solution of the large system

$$\mathcal{M}x = b, \quad \text{where} \quad \mathcal{M} = \begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix}, \qquad A = A^T$$

with B full column rank. The problem size makes the use of preconditioned iterative methods mandatory, where the preconditioner efficiently exploits the block structure of $\mathcal{M}$. In this context, the preconditioners

$$P_1 = \begin{bmatrix} I & B \\ B^T & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} A & B \\ 0 & -B^T A^{-1} B \end{bmatrix}$$

are particularly well suited; see e.g. [2,3,4]. However, their application involves solving with $B^T B$ and/or A at each iteration. On large problems this is infeasible, thus these solves are replaced by cheaper operators [1], leading to inexact preconditioners. In this talk we describe some spectral properties of the inexactly preconditioned system matrix, and their effect on the convergence of minimum residual Krylov subspace solvers.

References

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