FLEXIBLE AND INEXACT KRYLOV SUBSPACE METHODS

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Abstract

Flexible Krylov methods refers to a class of methods which accept variable (or flexible) preconditioning, i.e., preconditioning that can change from one step to the next. In other words, given a Krylov subspace method, such as CG, GMRES, QMR, etc. for the solution of a linear system $Ax = b$, instead of having a fixed preconditioner M and the (right) preconditioned equation $AM^{-1}y = b$ $(Mx = y)$, one may have a different matrix, say M_i at each step. In this presentation, we study the case where the preconditioner itself is a Krylov subspace method.

There are several papers in the literature where such situation is presented and numerical examples given. For example, GMRES is used as a preconditioner for FGMRES [Saad, Chapman and Saad], or QMR as the preconditioner for FQMR [Szyld and Vogel]. A general theory is presented encompassing these two cases, and many others. In fact, our general theory applies to any outer Krylov method with any inner one. Truncated methods are included in our theory as well.

The overall space where the minimization or Galerkin condition is imposed is no longer a Krylov subspace, but instead a subspace of a larger Krylov space. We show how this subspace keeps growing as the outer iteration progresses, thus providing a convergence theory for these inner-outer methods. One of our goals is to show that these inner-outer methods are very competitive. Experiments with Flexible truncated GMRES, in which the same amount of storage is used as GMRES(m) illustrate the advantage of the inner-outer methods.

Inexact Krylov subspace methods refer to the case where the matrix–vector product is not performed exactly. At the kth step of a Krylov subspace method the action Av is replaced with $(A + E_k)v$ where E_k is some error. In a series of CERFACS reports in 2000, Bouras, Frayssé and Giraud demonstrate experimentally that as the iteration progresses, i.e, as k increases, $\|E_k\|$ may be allowed to grow. In the second part of our presentation we provide a general framework for the understanding of these Inexact Krylov subspace methods, and in particular explain the theory behind the experiments reported in the literature. Furthermore, assuming exact arithmetic, our analysis produces computable criteria to bound the inexactness of the matrix–vector multiplication, i.e., $\|E_k\|$, in such a way as to maintain the convergence of the Krylov subspace method. The theory developed is applied to several problems including the solution of Schur complement systems, linear systems which depend on a parameter, and eigenvalue problems. Numerical experiments for some of these scientific applications are reported, where the computable criteria are successfully applied.