Applications of Linear Solvers to Image Reconstruction from Limited-Data

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The issue of image reconstruction from limited-data is present basically in geophysical tomography (geotomography). This is a non-destructive method which maps the interior structure of a cross-borehole section using a technique of remote-sensing measurements. The method consists of two stages: a process of data collection and data processing for obtaining an image of the desired structure. The data processing is called the image reconstruction.

In the process of data collection, the quantity of an information carrier (electrical currents, seismic or electromagnetic waves) passing through the examined object along multiple straight-line rays is measured. The rays are traced between the information carrier source mowing up and down along the one borehole and measurement points spacing along the second borehole. For acquiring the sufficient information on the interior of the object, the rays should have various angular orientations. Unfortunately, the object in geophysical tomography can be only accessible for measurements from two edges. It means that this measurement technique provides limited-data, that is, the information on the interior of the object is incomplete.

In the process of image reconstruction, the information included in the data is processed to obtain an image of some feature representing the interior structure. For the limited-data, a forward model, which describes a relation between the space of image and the space of measurements, should be presented in a discrete form. It implies that each ray is described by a linear equation and a system of such equations is formed for the examined object.

Similarly as in many applications of tomography, image reconstruction in geotomography boils down to the solution of a system of linear equations. The system is large, usually overdetermined, sparse, unsymmetrical and ill-conditioned. Moreover, such a measurement technique entails the rank-deficient system and thus a regularisation is necessary for obtaining the desired solution. Many linear solvers and many regularisation techniques have been proposed to solve such systems of equations. However, because of geotomography's specific features, e.g. very noisy data and a limited angular range of rays, some linear solvers frequently-used for solving PDE problems or for image reconstruction in other tomography applications cannot be applied here.

The aim of this paper is:

- to test the usefulness of chosen linear solvers for image reconstruction from limited-data,
- to propose modifications of some methods in order to increase their effectiveness.

The tests were carried out for the following methods: the Tikhonov regularisation, the SVD method, the Landweber iterations, the EM-ML method, the EM-MAP method, the CG method, the Lanczos method, the LSQR method, the GMRES method, the QMR method, the CGS method. In the case of iterative methods, the regularisation has been usually achieved by an early-termination of iterations. For the non-iterative methods, the regularisation parameters have been evaluated by the L-curve or OCV criteria.

The usefulness of the solvers is determined on the basis of a multicriterial estimation of images reconstructed from noise-free and noisy data generated from original images. The noise level is estimated statistically from real data. As a result, the Expectation-Maximisation (EM) method based on Maximum A Posteriori (MAP) approach has been chosen as the most effective for the purpose. The best results have been obtained for the MAP estimates with the Gibbs statistics in which weighted factors between "cliques" of pixels have been object-depended.