FINITE PRECISION KRYLOV METHODS IN A NUTSHELL

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Abstract

This contribution is concerned with a unified approach to finite precision Krylov subspace methods. The basic methods, like the methods of Arnoldi and Lanczos and the CG method, are presented in a common framework chosen to fit the analysis of their behaviour in finite precision.

The approach is based on the analysis of perturbed equations of type

$$AQ_k = Q_{k+1}\underline{C}_k = Q_k C_k + q_{k+1} c_{k+1,k} e_k^T,$$

where $A \in \mathbb{K}^{n \times n}$, $Q_k \in \mathbb{K}^{n \times k}$, $C_k \in \mathbb{K}^{k \times k}$ and $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. In most of the methods discussed C_k is Hessenberg.

We unify and generalize the core of existing error analyses and give a new approach based on properties of the matrix C_k. This new approach is *not* useful in computing bounds, but eases understanding and may help in the development of new algorithms. It is based on previous work [1].

References

[1] Jens–Peter M. Zemke. How Orthogonality is Lost in Krylov Methods. 2001, in G. Alefeld et.al., editors, Symbolic Algebraic Methods and Verification Methods, pages 255–266, Springer, Wien, New York.