NUMERICAL BEHAVIOUR OF HIGHAM'S SCALED METHOD FOR POLAR DECOMPOSITION

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Abstract

This talk is a joint work with Andrzej Kiełbasiński (University of Warsaw, Poland). We present rounding error analysis of Higham's method [1] for computing the unitary factor U from the polar decomposition of a complex nonsingular matrix A, $A = UH$, U - unitary, H - Hermitian positive-definite. In the scaled method of Higham, one constructs a sequence

$$X_{k+1} = \frac{1}{2}\Big(\gamma_k X_k + \frac{1}{\gamma_k} X^{-H}\Big), \quad X_0 = A,$$

convergent to U. It is easy to verify that U is a common unitary factor of all matrices $X_k, k = 0, 1, \ldots$. This fact is used in the rounding error analysis, presented in the talk (see [2]).

Fast reduction of large condition numbers $\mathrm{cond}(X_k)$ of X_k is the most important advantage of Higham's method. Frequent appearances of scaling parameters γ_k distinctly smaller than the optimal parameters of Higham [1] would deteriorate the quality of the computed results (simple counter-measures are proposed).

The unsatisfactory quality of some computed inverses is the main reason of the eventual failure in computing acceptable polar factors U and H. However, it follows from numerical experiments, done by Paweł Zieliński (High School of Engineering, Opole, Poland), that the inversion of X_k via *qr*-factorization with column pivoting (instead of the triangular one) practicaly removes this source of failures (see [3]).

References

[1] N.J. Higham. Computing the polar decomposition - with applications, *SIAM J. Sci. Stat. Comput.* 7:1160–1174 (1986).

[2] A. Kiełbasiński, K. Ziętak. Numerical behaviour of Higham's scaled method for computing polar decomposition, in preparation.

[3] A. Kiełbasiński, P. Zieliński, K. Ziętak. Numerical experiments with Higham's scaled method for polar decomposition, in preparation.