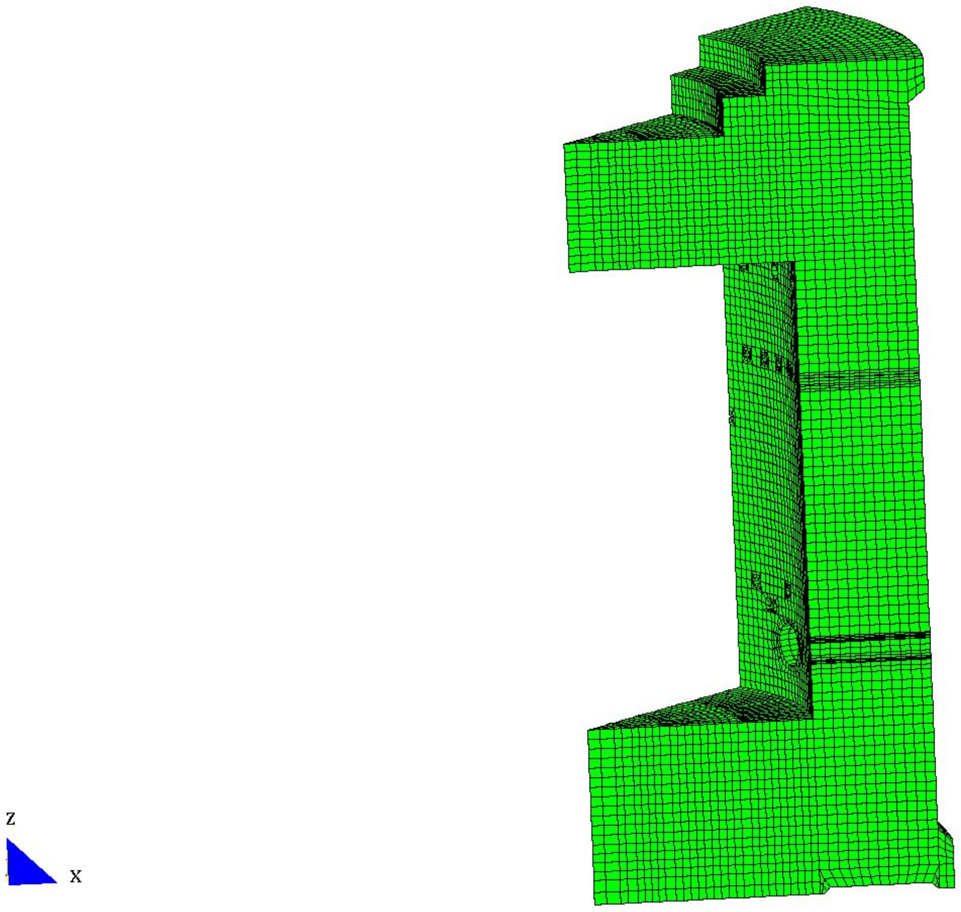
Czech Technical University in Prague
Faculty of Civil Engineering
Department of Mechanics

Analysis of Reactor Vessel by Domain Decomposition Methods

Jaroslav Kruis

Outline

- mechanical, thermal, moisture and coupled analyses
- domain decomposition methods
- analysis of vessel


z
x

Creep Analysis

governing equation

$$\boldsymbol{K}\dot{\boldsymbol{d}} = \dot{\boldsymbol{f}} + \int_V \boldsymbol{B}^T \boldsymbol{D} \dot{\boldsymbol{\varepsilon}}_{ir} \, \mathrm{d}V$$

material model is based on Bažant’s B3 creep model

Hydro-Thermo Analysis

system of governing equations

$$
\begin{pmatrix} \boldsymbol{C}_{TT} & \boldsymbol{C}_{Tp_1} & \boldsymbol{C}_{Tp_2} \\ \boldsymbol{C}_{p_1T} & \boldsymbol{C}_{p_1p_1} & \boldsymbol{C}_{p_1p_2} \\ \boldsymbol{C}_{p_2T} & \boldsymbol{C}_{p_2p_1} & \boldsymbol{C}_{p_2p_2} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{d}}_T \\ \dot{\boldsymbol{d}}_{p_1} \\ \dot{\boldsymbol{d}}_{p_1} \end{pmatrix} +
$$

$$
\begin{pmatrix} \boldsymbol{K}_{TT} & \boldsymbol{K}_{Tp_1} & \boldsymbol{K}_{Tp_2} \\ \boldsymbol{K}_{p_1T} & \boldsymbol{K}_{p_1p_1} & \boldsymbol{K}_{p_1p_2} \\ \boldsymbol{K}_{p_2T} & \boldsymbol{K}_{p_2p_1} & \boldsymbol{K}_{p_2p_2} \end{pmatrix} \begin{pmatrix} \boldsymbol{d}_T \\ \boldsymbol{d}_{p_1} \\ \boldsymbol{d}_{p_1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}_T \\ \boldsymbol{f}_{p_1} \\ \boldsymbol{f}_{p_1} \end{pmatrix}
$$

Hydro-Thermo-Mechanical Analysis

$$
\begin{pmatrix}
\boldsymbol{C}_{uu} & \boldsymbol{C}_{uT} & \boldsymbol{C}_{up_1} & \boldsymbol{C}_{up_2} \\
\boldsymbol{C}_{Tu} & \boldsymbol{C}_{TT} & \boldsymbol{C}_{Tp_1} & \boldsymbol{C}_{Tp_2} \\
\boldsymbol{C}_{p_1u} & \boldsymbol{C}_{p_1T} & \boldsymbol{C}_{p_1p_1} & \boldsymbol{C}_{p_1p_2} \\
\boldsymbol{C}_{p_2u} & \boldsymbol{C}_{p_2T} & \boldsymbol{C}_{p_2p_1} & \boldsymbol{C}_{p_2p_2}
\end{pmatrix}
\begin{pmatrix}
\dot{\boldsymbol{d}}_u \\ \dot{\boldsymbol{d}}_T \\ \dot{\boldsymbol{d}}_{p_1} \\ \dot{\boldsymbol{d}}_{p_2}
\end{pmatrix} +
$$

$$
+ \begin{pmatrix}
\boldsymbol{K}_{uu} & \boldsymbol{K}_{uT} & \boldsymbol{K}_{up_1} & \boldsymbol{K}_{up_2} \\
\boldsymbol{K}_{Tu} & \boldsymbol{K}_{TT} & \boldsymbol{K}_{Tp_1} & \boldsymbol{K}_{Tp_2} \\
\boldsymbol{K}_{p_1u} & \boldsymbol{K}_{p_1T} & \boldsymbol{K}_{p_1p_1} & \boldsymbol{K}_{p_1p_2} \\
\boldsymbol{K}_{p_2u} & \boldsymbol{K}_{p_2T} & \boldsymbol{K}_{p_2p_1} & \boldsymbol{K}_{p_2p_2}
\end{pmatrix}
\begin{pmatrix}
\boldsymbol{d}_u \\ \boldsymbol{d}_T \\ \boldsymbol{d}_{p_1} \\ \boldsymbol{d}_{p_2}
\end{pmatrix} =
\begin{pmatrix}
\boldsymbol{f}_u \\ \boldsymbol{f}_T \\ \boldsymbol{f}_{p_1} \\ \boldsymbol{f}_{p_2}
\end{pmatrix}
$$

$$\boldsymbol{C}(\boldsymbol{d})\dot{\boldsymbol{d}} + \boldsymbol{K}(\boldsymbol{d})\boldsymbol{d} = \boldsymbol{f}$$

$$\boldsymbol{d}_{n+1} = \boldsymbol{d}_n + \Delta t \boldsymbol{d}_{n+\alpha}$$

$$\boldsymbol{v}_{n+\alpha} = (1-\alpha)\boldsymbol{v}_n + \alpha \boldsymbol{v}_{n+1}$$

$$\left(\boldsymbol{C}(\boldsymbol{d}) + \Delta t \alpha \boldsymbol{K}(\boldsymbol{d})\right)\boldsymbol{v}_{n+1} = \boldsymbol{f}_{n+1} - \boldsymbol{K}\left(\boldsymbol{d}_n + \Delta t(1-\alpha)\boldsymbol{v}_n\right)$$

Domain Decomposition Methods

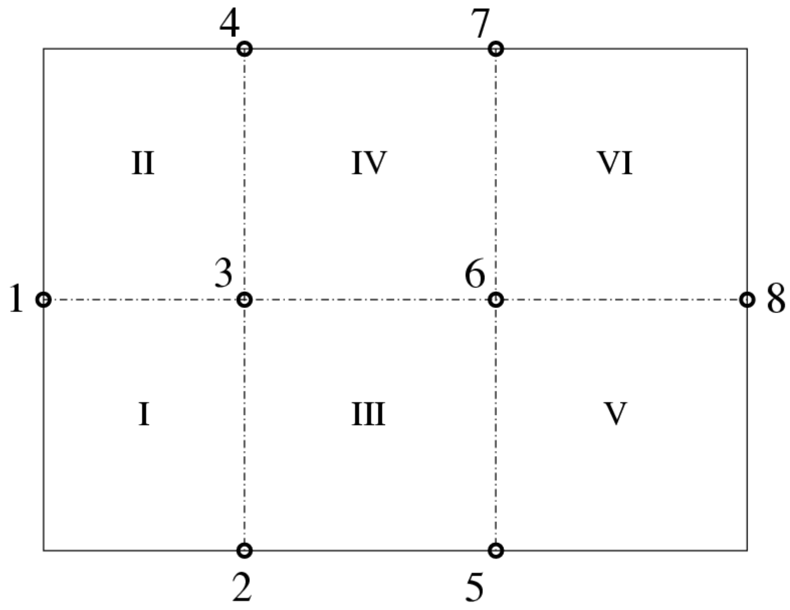
- Schur complement method (method of substructures)
- FETI-DP method

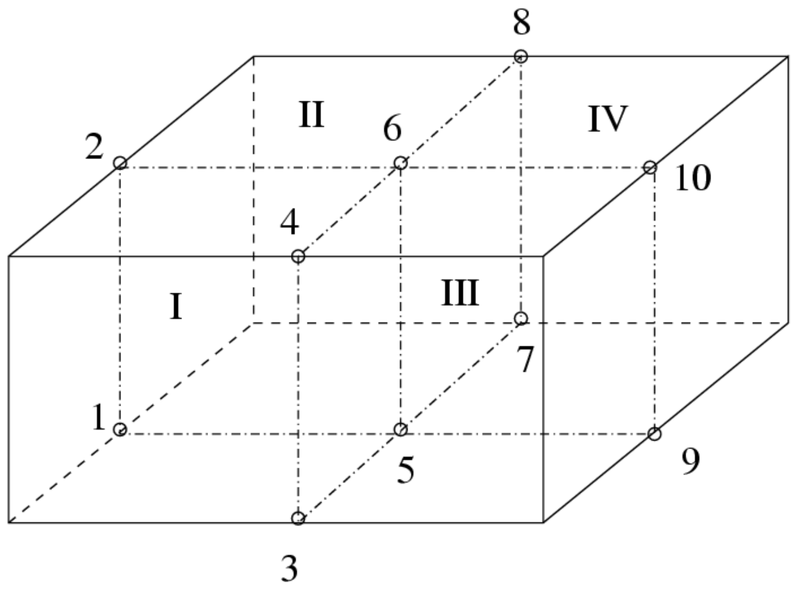
FETI-DP

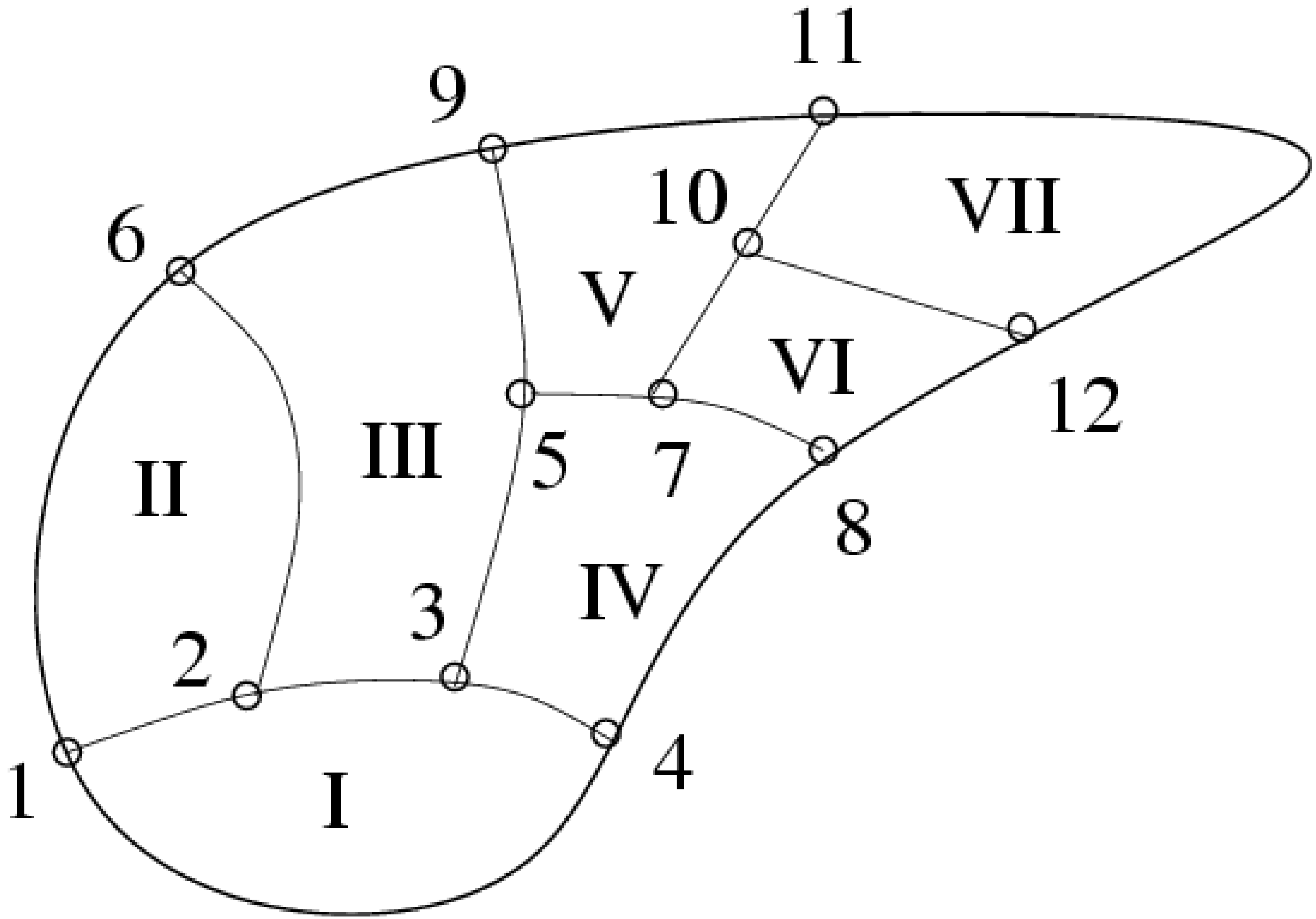
introduced by Farhat, Lesoinne, Pierson, LeTallec, Rixen (2000, 2001)

convergence studied by Mandel, Tezaur (2001)

- combination of the Schur complement method and the FETI method
- nodes (unknowns) are split into three groups
- the classical conjugate gradient method is used


4
7
II
IV
VI
1
3
6
8
I
III
V
2
5


8
II
6
IV
2
10
4
I
III
7
1
5
9
3


11
9
10
VII
6
V
VI
12
III
5
7
II
8
IV
3
2
4
1
I

nodal unknowns

$$\boldsymbol{u}^j = \begin{pmatrix} \boldsymbol{u}_i^j \\ \\ \boldsymbol{u}_b^j \\ \\ \boldsymbol{u}_c^j \end{pmatrix} = \begin{pmatrix} \boldsymbol{u}_r^j \\ \\ \boldsymbol{u}_c^j \end{pmatrix} = \boldsymbol{G}^j \boldsymbol{u}_r^j + \tilde{\boldsymbol{B}}_c^j \boldsymbol{u}_c$$

$$\boldsymbol{G}^j = \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{0} \end{pmatrix} \qquad \tilde{\boldsymbol{B}}_c^j = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{B}_c^j \end{pmatrix}$$

$$\boldsymbol{u}_c^j = \boldsymbol{B}_c^j \boldsymbol{u}_c$$

$$\boldsymbol{K}^j = \begin{pmatrix} \boldsymbol{K}^j_{rr} & \boldsymbol{K}^j_{rc} \\ \boldsymbol{K}^j_{cr} & \boldsymbol{K}^j_{cc} \end{pmatrix}$$

$$\boldsymbol{f}^j = \begin{pmatrix} \boldsymbol{f}^j_r \\ \boldsymbol{f}^j_c \end{pmatrix}$$

$$\boldsymbol{B}^j_r \boldsymbol{u}^j_r + \boldsymbol{B}^k_r \boldsymbol{u}^k_r = \boldsymbol{0}$$

$$\sum_{j=1}^{m} \boldsymbol{B}^j_r \boldsymbol{u}^j_r = \boldsymbol{0}$$

$$\Pi = \sum_{j=1}^{m} \left(\frac{1}{2} (\boldsymbol{u}^j)^T \boldsymbol{K}^j \boldsymbol{u}^j - (\boldsymbol{u}^j)^T \boldsymbol{f}^j + \boldsymbol{\lambda}^T \boldsymbol{B}_r^j \boldsymbol{u}_r^j \right)$$

$$\begin{aligned}
\Pi &= \sum_{j=1}^{m} \left(\frac{1}{2} (\boldsymbol{u}_r^j)^T (\boldsymbol{G}^j)^T \boldsymbol{K}^j \boldsymbol{G}^j \boldsymbol{u}_r^j + \frac{1}{2} (\boldsymbol{u}_r^j)^T (\boldsymbol{G}^j)^T \boldsymbol{K}^j \tilde{\boldsymbol{B}}_c^j \boldsymbol{u}_c \right. \\
&+ \frac{1}{2} (\boldsymbol{u}_c)^T (\tilde{\boldsymbol{B}}_c^j)^T \boldsymbol{K}^j \boldsymbol{G}^j \boldsymbol{u}_r^j + \frac{1}{2} (\boldsymbol{u}_c)^T (\tilde{\boldsymbol{B}}_c^j)^T \boldsymbol{K}^j \tilde{\boldsymbol{B}}_c^j \boldsymbol{u}_c \\
&- \left. (\boldsymbol{u}_r^j)^T (\boldsymbol{G}^j)^T \boldsymbol{f}^j - (\boldsymbol{u}_c)^T (\tilde{\boldsymbol{B}}_c^j)^T \boldsymbol{f}^j + \boldsymbol{\lambda}^T \boldsymbol{B}_r^j \boldsymbol{u}_r^j \right)
\end{aligned}$$

$$\begin{aligned}\Pi &= \sum_{j=1}^{m}\left(\frac{1}{2}(\boldsymbol{u}_r^j)^T(\boldsymbol{G}^j)^T\boldsymbol{K}^j\boldsymbol{G}^j\boldsymbol{u}_r^j + \frac{1}{2}(\boldsymbol{u}_r^j)^T(\boldsymbol{G}^j)^T\boldsymbol{K}^j\tilde{\boldsymbol{B}}_c^j\boldsymbol{u}_c\right.\\
&+ \frac{1}{2}(\boldsymbol{u}_c)^T(\tilde{\boldsymbol{B}}_c^j)^T\boldsymbol{K}^j\boldsymbol{G}^j\boldsymbol{u}_r^j + \frac{1}{2}(\boldsymbol{u}_c)^T(\tilde{\boldsymbol{B}}_c^j)^T\boldsymbol{K}^j\tilde{\boldsymbol{B}}_c^j\boldsymbol{u}_c\\
&- \left.(\boldsymbol{u}_r^j)^T(\boldsymbol{G}^j)^T\boldsymbol{f}^j - (\boldsymbol{u}_c)^T(\tilde{\boldsymbol{B}}_c^j)^T\boldsymbol{f}^j + \boldsymbol{\lambda}^T\boldsymbol{B}_r^j\boldsymbol{u}_r^j\right)\end{aligned}$$

$$\begin{aligned}\Pi &= \sum_{j=1}^{m}\left(\frac{1}{2}(\boldsymbol{u}_r^j)^T\boldsymbol{K}_{rr}^j\boldsymbol{u}_r^j + \frac{1}{2}(\boldsymbol{u}_r^j)^T\boldsymbol{K}_{rc}^j\boldsymbol{u}_c^j\right.\\
&+ \frac{1}{2}(\boldsymbol{u}_c^j)^T\boldsymbol{K}_{cr}^j\boldsymbol{u}_r^j + \frac{1}{2}(\boldsymbol{u}_c^j)^T\boldsymbol{K}_{cc}^j\boldsymbol{u}_c^j\\
&- \left.(\boldsymbol{u}_r^j)^T\boldsymbol{f}_r^j - (\boldsymbol{u}_c^j)^T\boldsymbol{f}_c^j + \boldsymbol{\lambda}^T\boldsymbol{B}_r^j\boldsymbol{u}_r^j\right)\end{aligned}$$

$$
\begin{aligned}
(\boldsymbol{G}^j)^T \boldsymbol{K}^j \boldsymbol{G}^j &= \boldsymbol{K}^j_{rr}\\
(\boldsymbol{G}^j)^T \boldsymbol{K}^j \tilde{\boldsymbol{B}}^j_c &= \boldsymbol{K}^j_{rc} \boldsymbol{B}^j_c\\
(\tilde{\boldsymbol{B}}^j_c)^T \boldsymbol{K}^j \boldsymbol{G}^j &= (\boldsymbol{B}^j_c)^T \boldsymbol{K}^j_{cr}\\
(\tilde{\boldsymbol{B}}^j_c)^T \boldsymbol{K}^j \tilde{\boldsymbol{B}}^j_c &= (\boldsymbol{B}^j_c)^T \boldsymbol{K}^j_{cc} \boldsymbol{B}^j_c
\end{aligned}
$$

$$
\begin{aligned}
(\boldsymbol{G}^j)^T \boldsymbol{f}^j &= \boldsymbol{f}^j_r\\
(\tilde{\boldsymbol{B}}^j_c)^T \boldsymbol{f}^j &= (\boldsymbol{B}^j_c)^T \boldsymbol{f}^j_c
\end{aligned}
$$

$$
\begin{aligned}
\Pi &= \sum_{j=1}^{m} \left(\frac{1}{2} (\boldsymbol{u}_r^j)^T \boldsymbol{K}_{rr}^j \boldsymbol{u}_r^j + \frac{1}{2} (\boldsymbol{u}_r^j)^T \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c \right. \\
&+ \frac{1}{2} (\boldsymbol{u}_c)^T (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j \boldsymbol{u}_r^j + \frac{1}{2} (\boldsymbol{u}_c)^T (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c \\
&- \left. (\boldsymbol{u}_r^j)^T \boldsymbol{f}_r^j - (\boldsymbol{u}_c)^T (\boldsymbol{B}_c^j)^T \boldsymbol{f}_c^j + \boldsymbol{\lambda}^T \boldsymbol{B}_r^j \boldsymbol{u}_r^j \right)
\end{aligned}
$$

$$\frac{\partial \Pi}{\partial \boldsymbol{u}_r^j} = \boldsymbol{K}_{rr}^j \boldsymbol{u}_r^j + \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c - \boldsymbol{f}_r^j + (\boldsymbol{B}_r^j)^T \boldsymbol{\lambda} = \boldsymbol{0}$$

$$\frac{\partial \Pi}{\partial \boldsymbol{u}_c} = \sum_{j=1}^{m} \left((\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j \boldsymbol{u}_r^j + (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c - (\boldsymbol{B}_c^j)^T \boldsymbol{f}_c^j \right) = \boldsymbol{0}$$

$$\frac{\partial \Pi}{\partial \boldsymbol{\lambda}} = \sum_{j=1}^{m} \boldsymbol{B}_r^j \boldsymbol{u}_r^j = \boldsymbol{0}$$

$$\frac{\partial \Pi}{\partial \boldsymbol{u}_r^j} = \boldsymbol{K}_{rr}^j \boldsymbol{u}_r^j + \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c - \boldsymbol{f}_r^j + (\boldsymbol{B}_r^j)^T \boldsymbol{\lambda} = \boldsymbol{0}$$

$$\boldsymbol{u}_r^j = (\boldsymbol{K}_{rr}^j)^{-1} \left(\boldsymbol{f}_r^j - (\boldsymbol{B}_r^j)^T \boldsymbol{\lambda} - \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c \right)$$

$$\frac{\partial \Pi}{\partial \boldsymbol{u}_c} = \sum_{j=1}^{m} \left((\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j \boldsymbol{u}_r^j + (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c - (\boldsymbol{B}_c^j)^T \boldsymbol{f}_c^j \right) = \boldsymbol{0}$$

$$\sum_{j=1}^{m} \Big(-(\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} (\boldsymbol{B}_r^j)^T \boldsymbol{\lambda} - (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c$$

$$+ (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{f}_r^j + (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c - (\boldsymbol{B}_c^j)^T \boldsymbol{f}_c^j \Big) = \boldsymbol{0}$$

$$\frac{\partial \Pi}{\partial \boldsymbol{\lambda}} = \sum_{j=1}^{m} \boldsymbol{B}_r^j \boldsymbol{u}_r^j = \boldsymbol{0}$$

$$\sum_{j=1}^{m} \Big(\boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{f}_r^j - \boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} (\boldsymbol{B}_r^j)^T \boldsymbol{\lambda}$$

$$- \boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \boldsymbol{u}_c \Big) = \boldsymbol{0}$$

$$
\begin{aligned}
\boldsymbol{S}_{cc} &= \sum_{j=1}^{m} (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cc}^j \boldsymbol{B}_c^j - \sum_{j=1}^{m} (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \\
\boldsymbol{F}_{cr} &= \sum_{j=1}^{m} (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} (\boldsymbol{B}_r^j)^T \\
\boldsymbol{F}_{rc} &= \sum_{j=1}^{m} \boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{K}_{rc}^j \boldsymbol{B}_c^j \\
\boldsymbol{F}_{rr} &= \sum_{j=1}^{m} \boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} (\boldsymbol{B}_r^j)^T
\end{aligned}
$$

$$\begin{aligned} \boldsymbol{s} &= \sum_{j=1}^{m} (\boldsymbol{B}_c^j)^T \boldsymbol{f}_c^j - \sum_{j=1}^{m} (\boldsymbol{B}_c^j)^T \boldsymbol{K}_{cr}^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{f}_r^j \\ \boldsymbol{g} &= \sum_{j=1}^{m} \boldsymbol{B}_r^j (\boldsymbol{K}_{rr}^j)^{-1} \boldsymbol{f}_r^j \end{aligned}$$

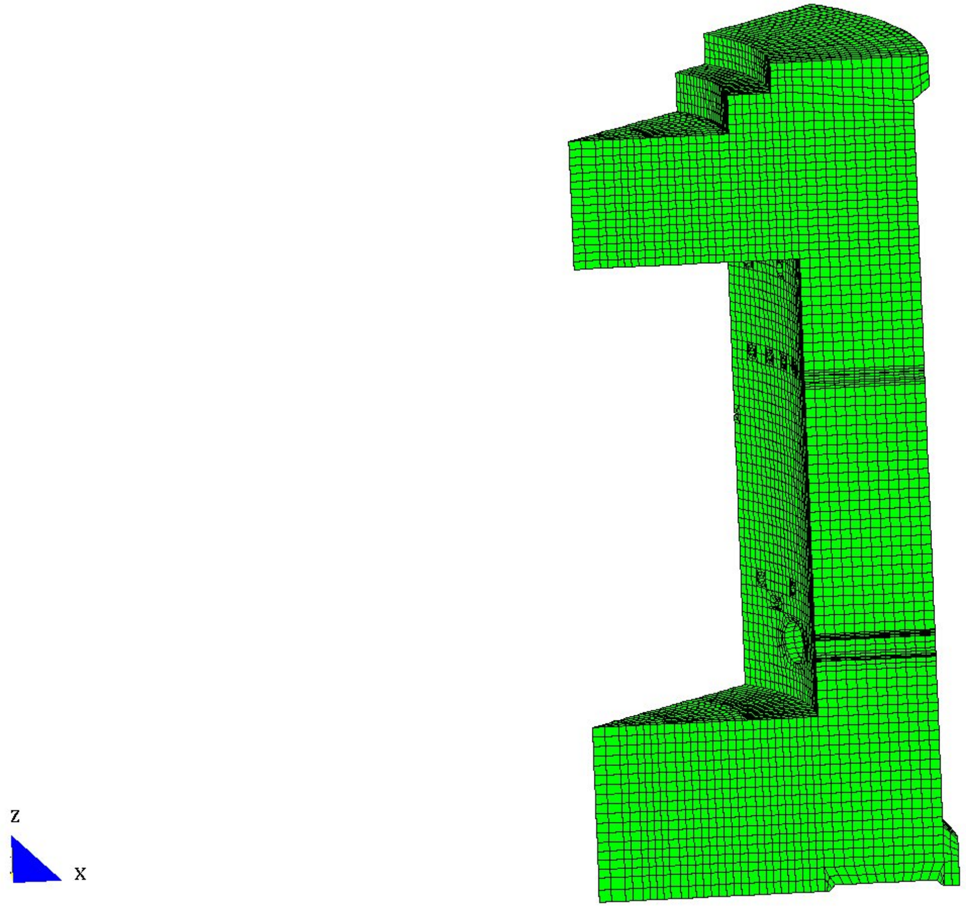
$$\begin{pmatrix} -\boldsymbol{S}_{cc} & \boldsymbol{F}_{cr} \\ \boldsymbol{F}_{rc} & \boldsymbol{F}_{rr} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_c \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -\boldsymbol{s} \\ \boldsymbol{g} \end{pmatrix}$$

$$\boldsymbol{u}_c = -\left(\boldsymbol{S}_{cc}\right)^{-1}\left(-\boldsymbol{s} - \boldsymbol{F}_{cr}\boldsymbol{\lambda}\right)$$

$$\left(\boldsymbol{F}_{rr} + \boldsymbol{F}_{rc}\left(\boldsymbol{S}_{cc}\right)^{-1}\boldsymbol{F}_{cr}\right)\boldsymbol{\lambda} = \boldsymbol{g} - \boldsymbol{F}_{rc}\left(\boldsymbol{S}_{cc}\right)^{-1}\boldsymbol{s}$$

Vessel Analysis

- prestressed concrete reactor vessel
- mechanical analysis
- heat and moisture transfer analysis
- coupled analysis
- more than 12,000 time steps (33 years)


Z
X

number of nodes	50,881
number of finite elements	44,486

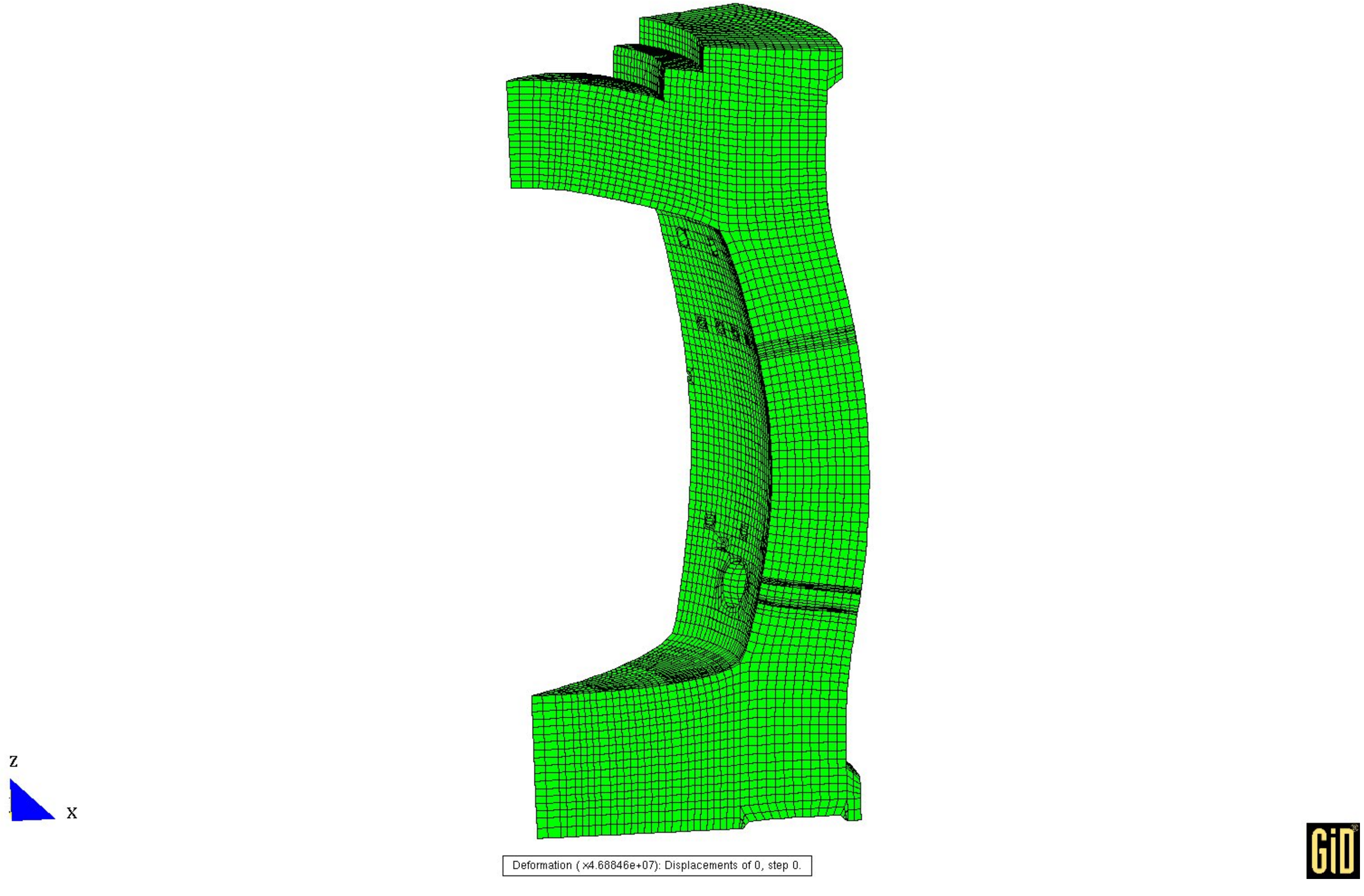
Available Hardware

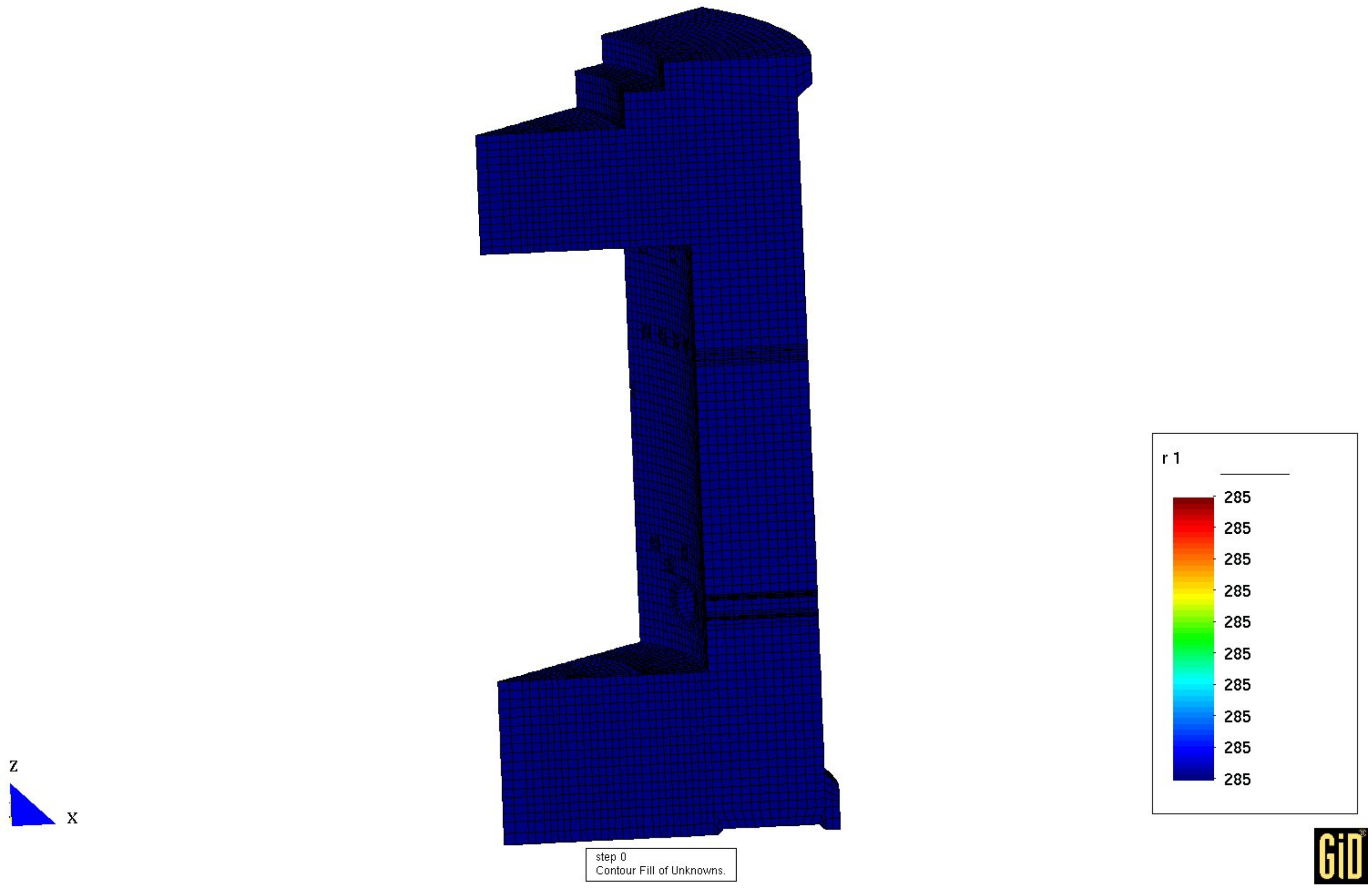
processor frequency	3.0 - 3.2 GHz
RAM	3.0 GB
number of processors	26

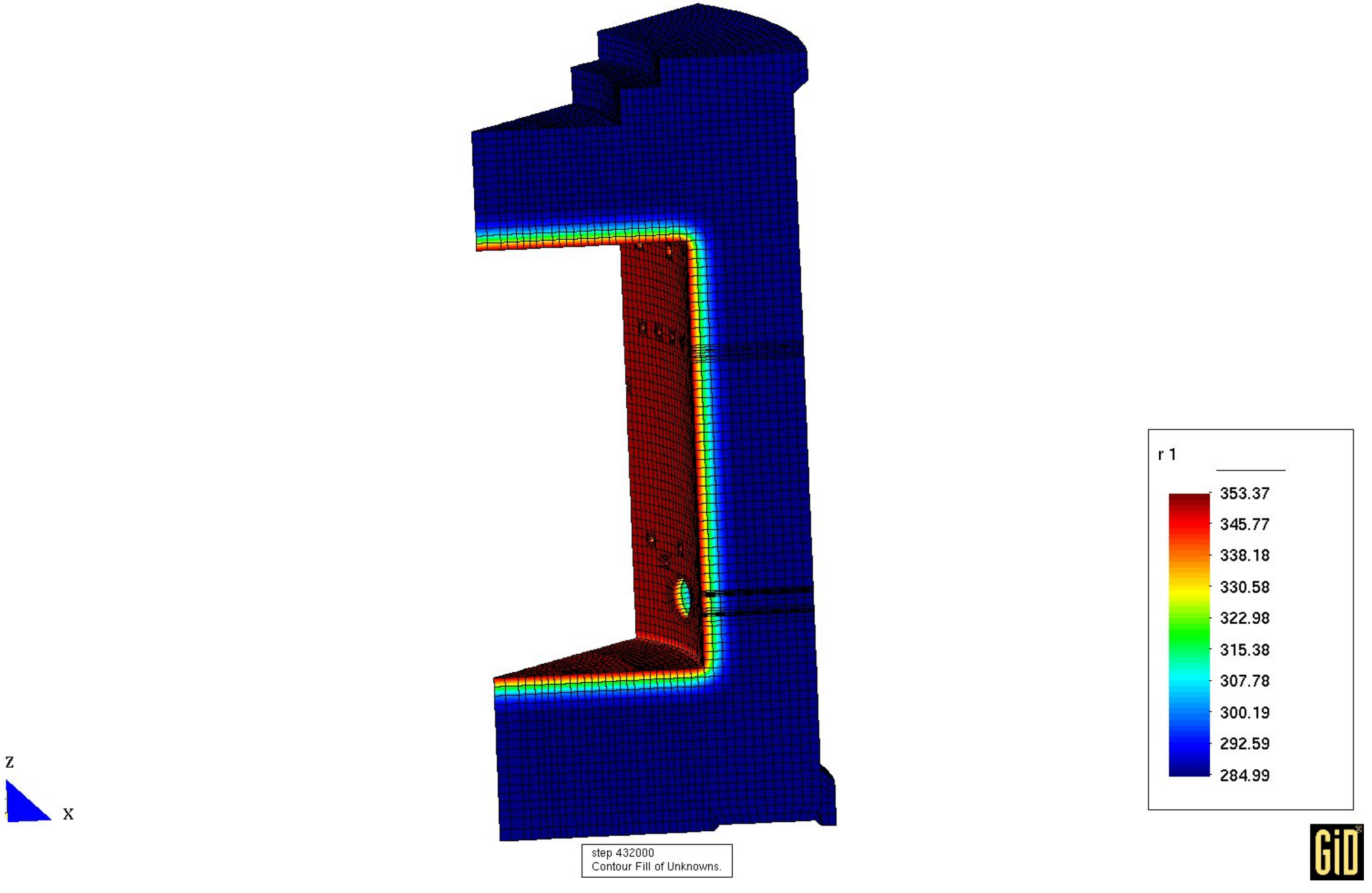
One-Processor Computing

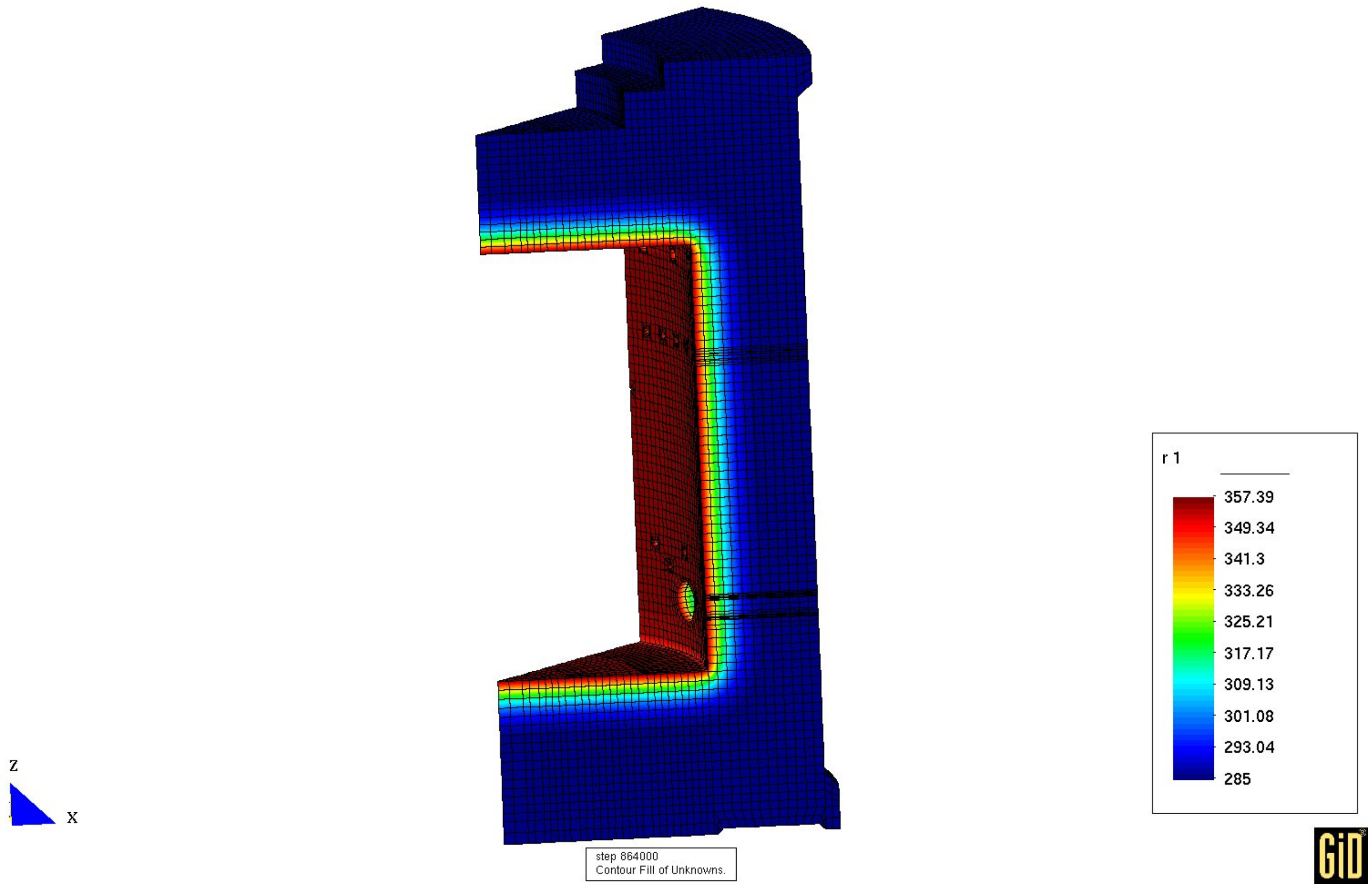
analysis	NDOFs	NEM		NI
mechanics	146,373	5,452,568	147,228,777	872
heat transfer	50,881	654,566	17,596,530	26

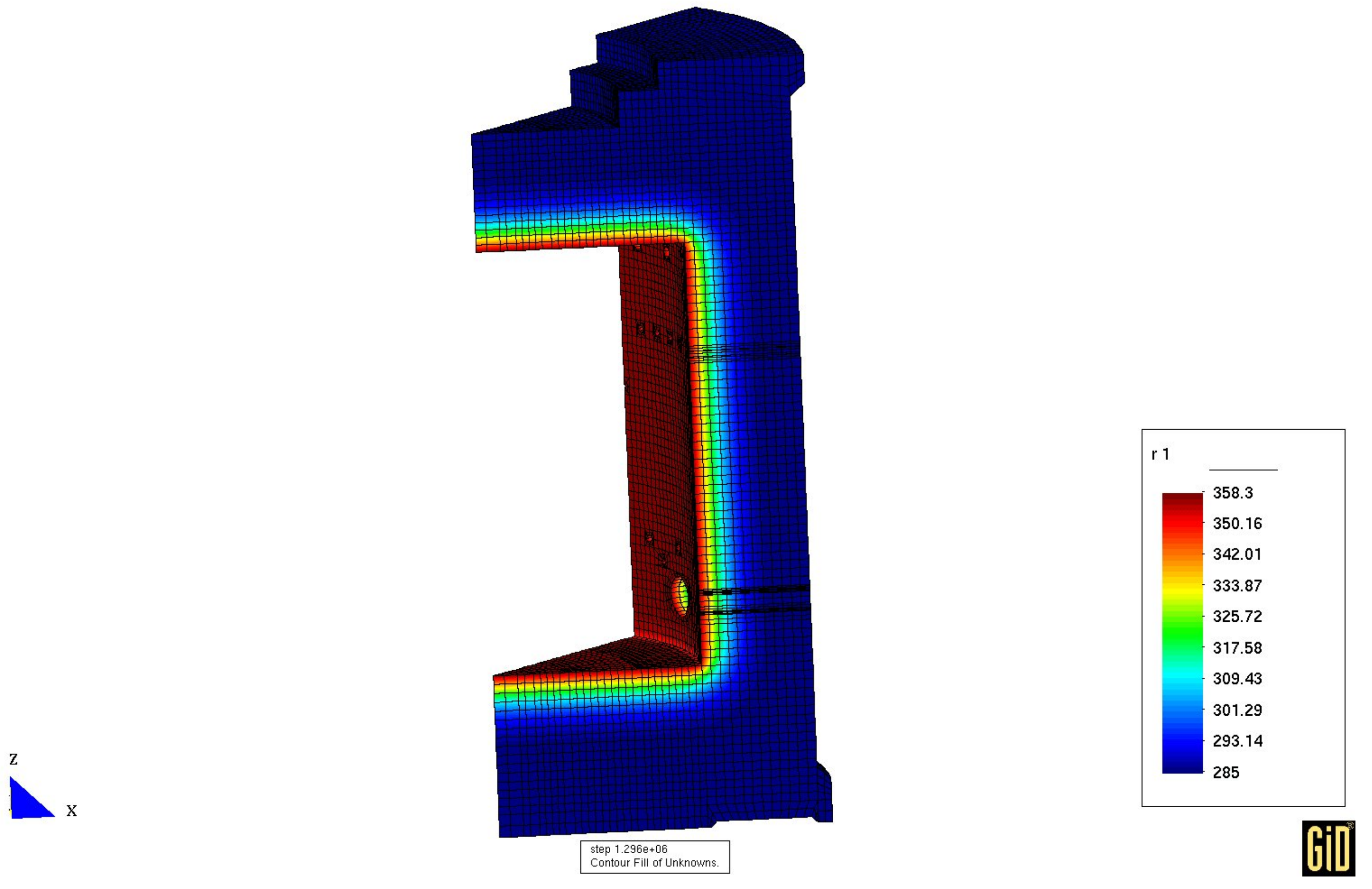
analysis	CG [s]	SD [s]	
mechanics	57.7	2	336.4
heat transfer	1.5	2	135.5

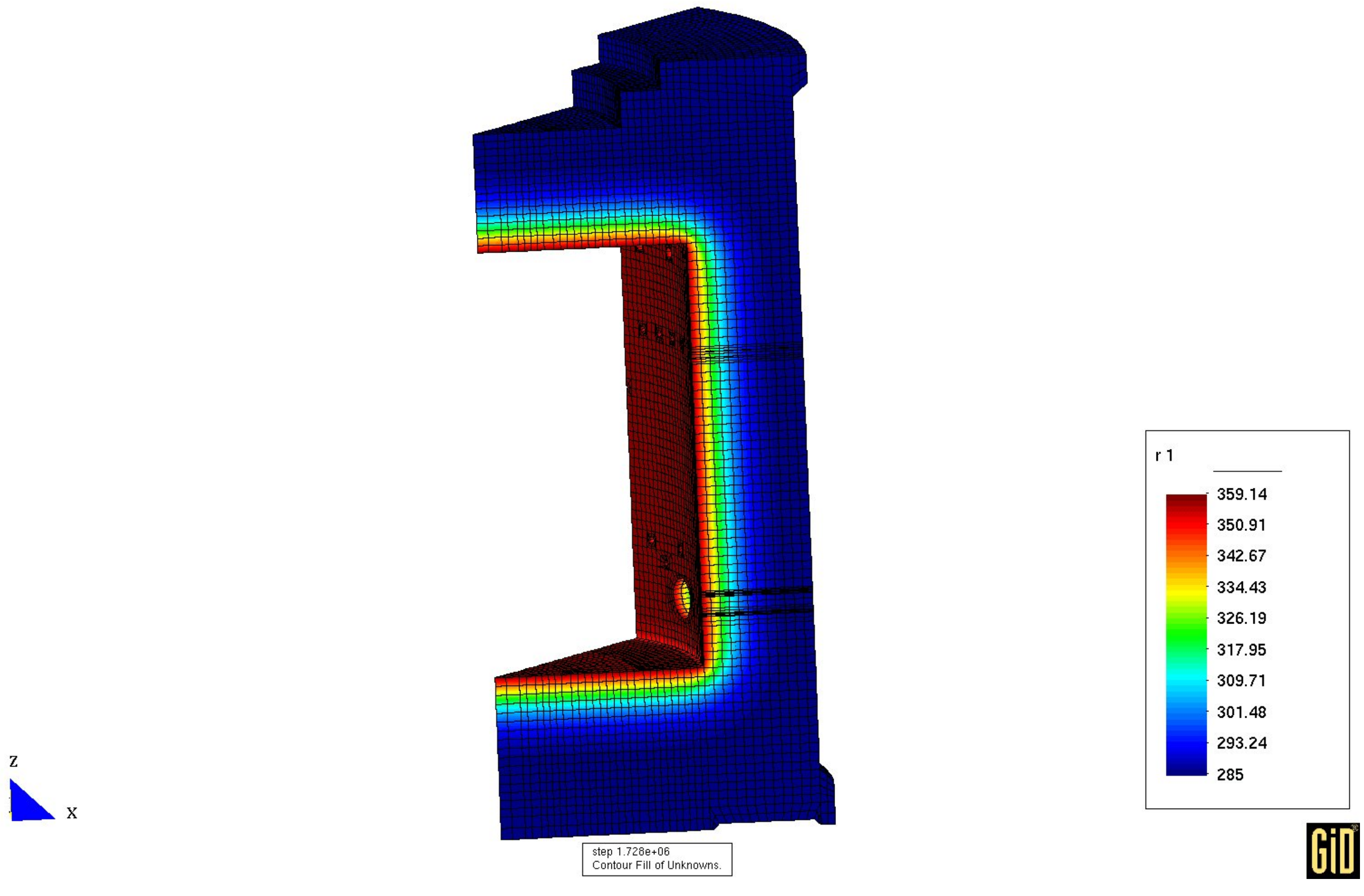

Z
X
Deformation (x4.68846e+07): Displacements of 0, step 0.
GiD

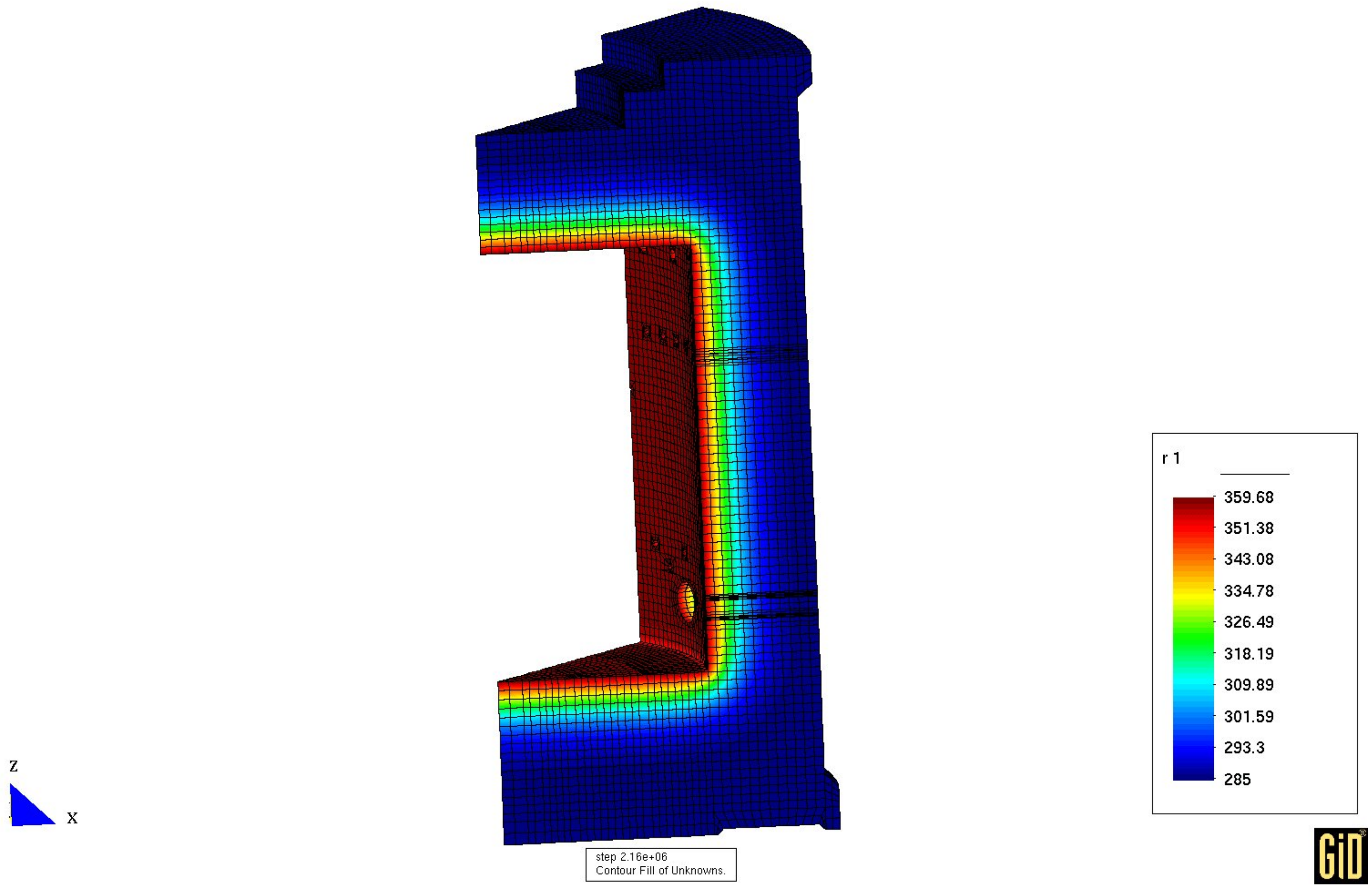

r 1
285
285
285
285
285
285
285
285
285
285
step 0
Contour Fill of Unknowns.
Z
X


z
x
step 432000
Contour Fill of Unknowns.
r 1
353.37
345.77
338.18
330.58
322.98
315.38
307.78
300.19
292.59
284.99

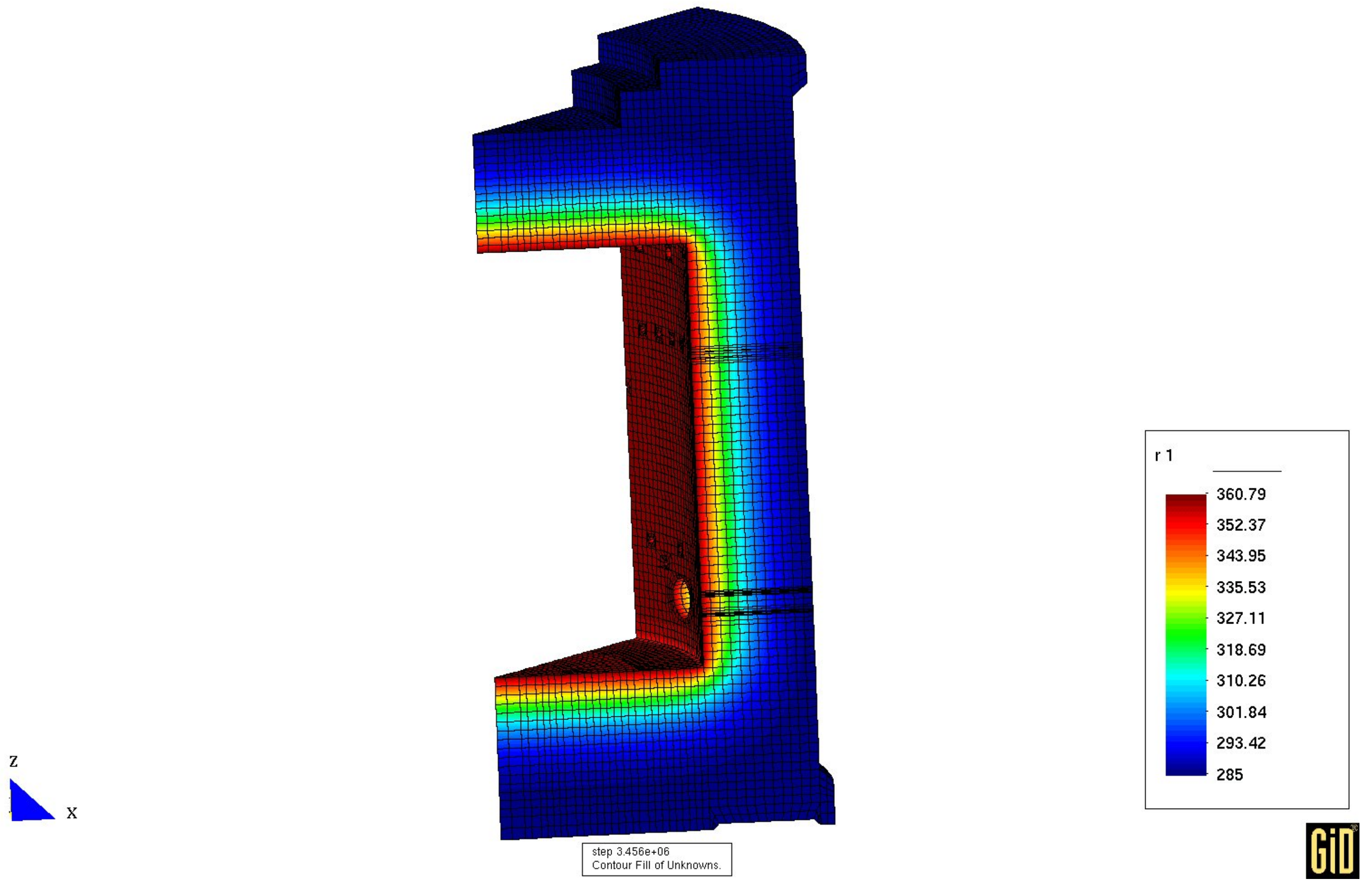

z
x
r 1
357.39
349.34
341.3
333.26
325.21
317.17
309.13
301.08
293.04
285
step 864000
Contour Fill of Unknowns.

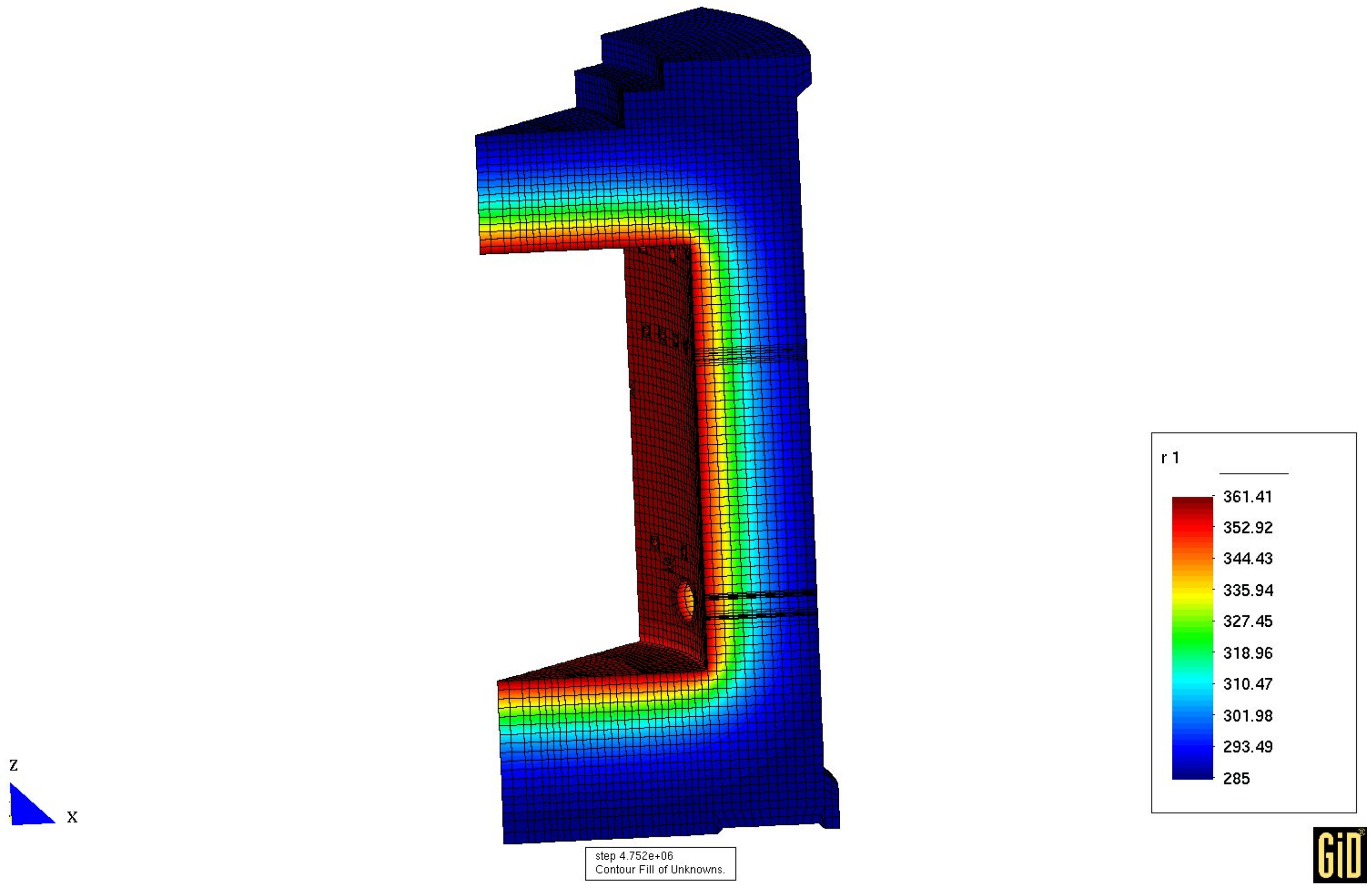

r 1
358.3
350.16
342.01
333.87
325.72
317.58
309.43
301.29
293.14
285
step 1.296e+06
Contour Fill of Unknowns.
z
x

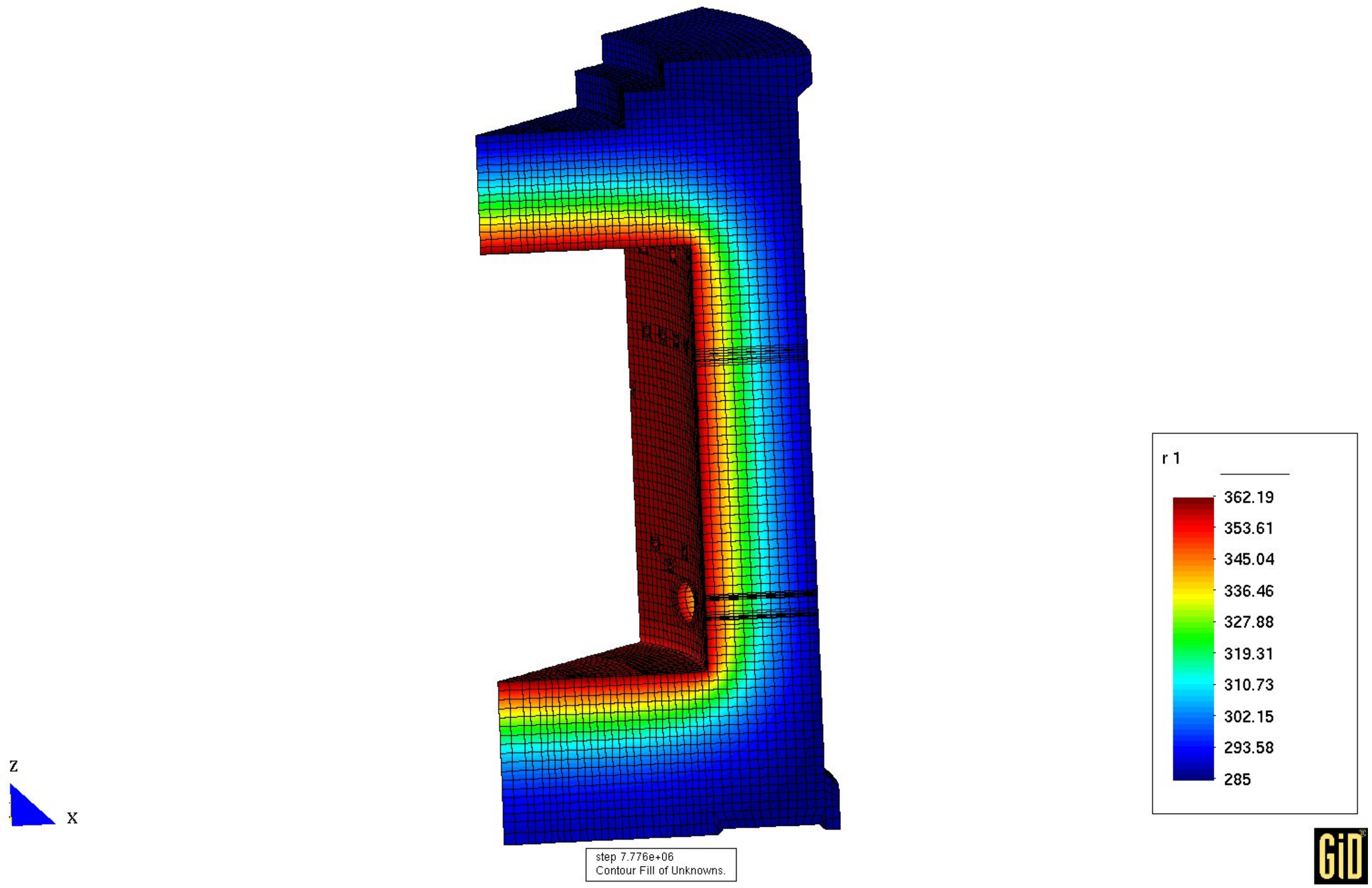

z
x
step 1.728e+06
Contour Fill of Unknowns.
r 1
359.14
350.91
342.67
334.43
326.19
317.95
309.71
301.48
293.24
285

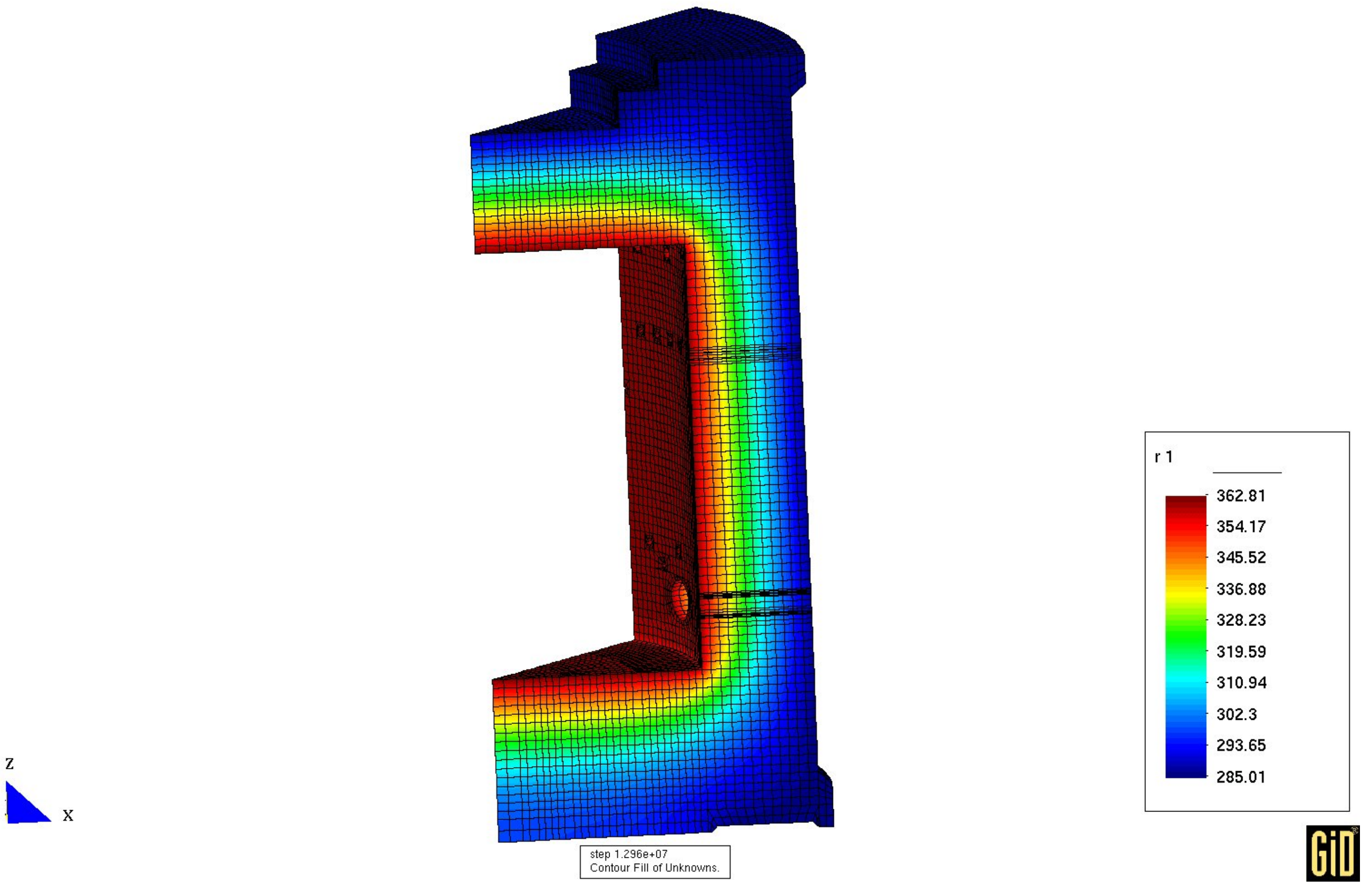


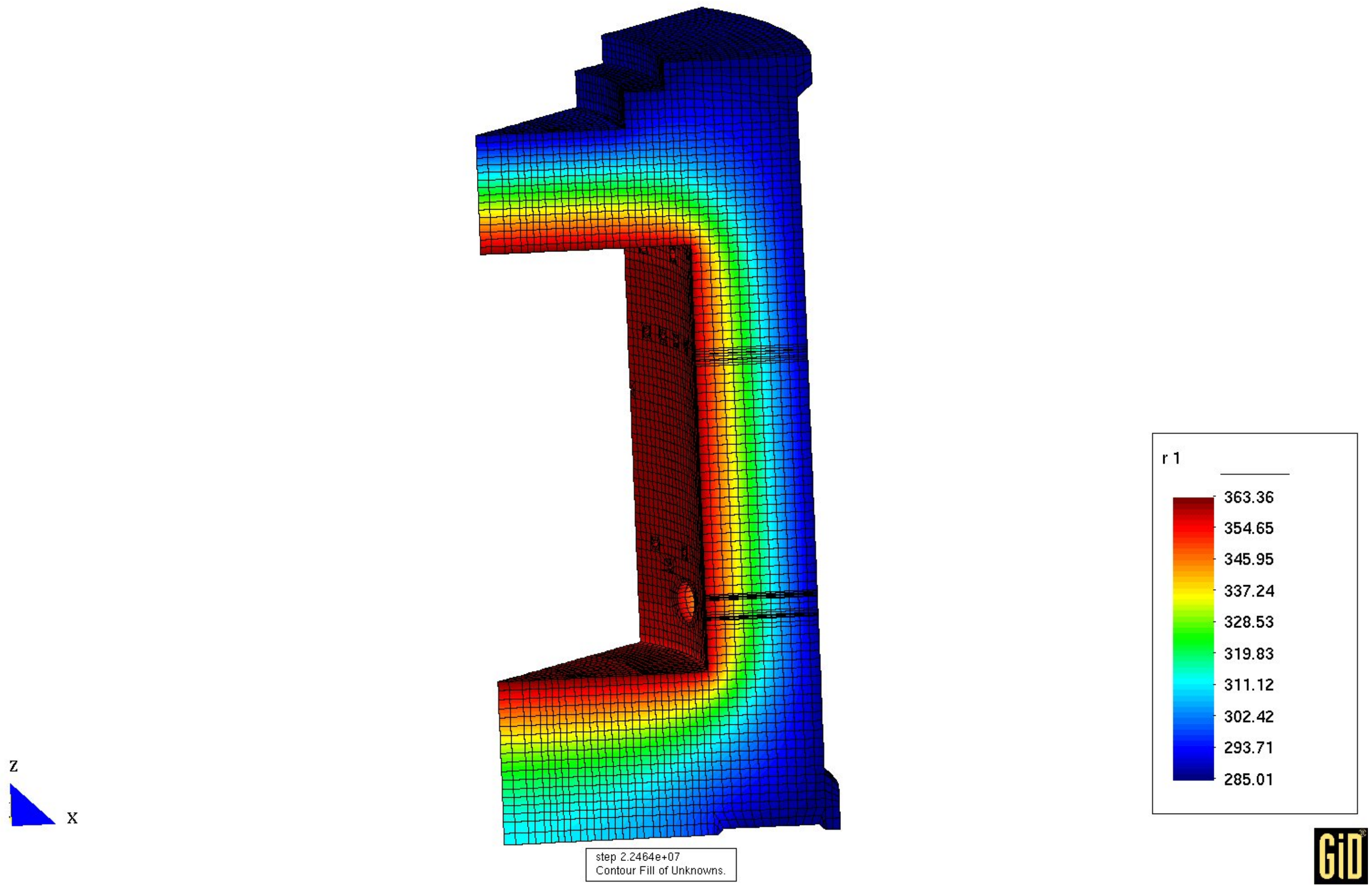
z
x
step 2.16e+06
Contour Fill of Unknowns.
r 1
359.68
351.38
343.08
334.78
326.49
318.19
309.89
301.59
293.3
285

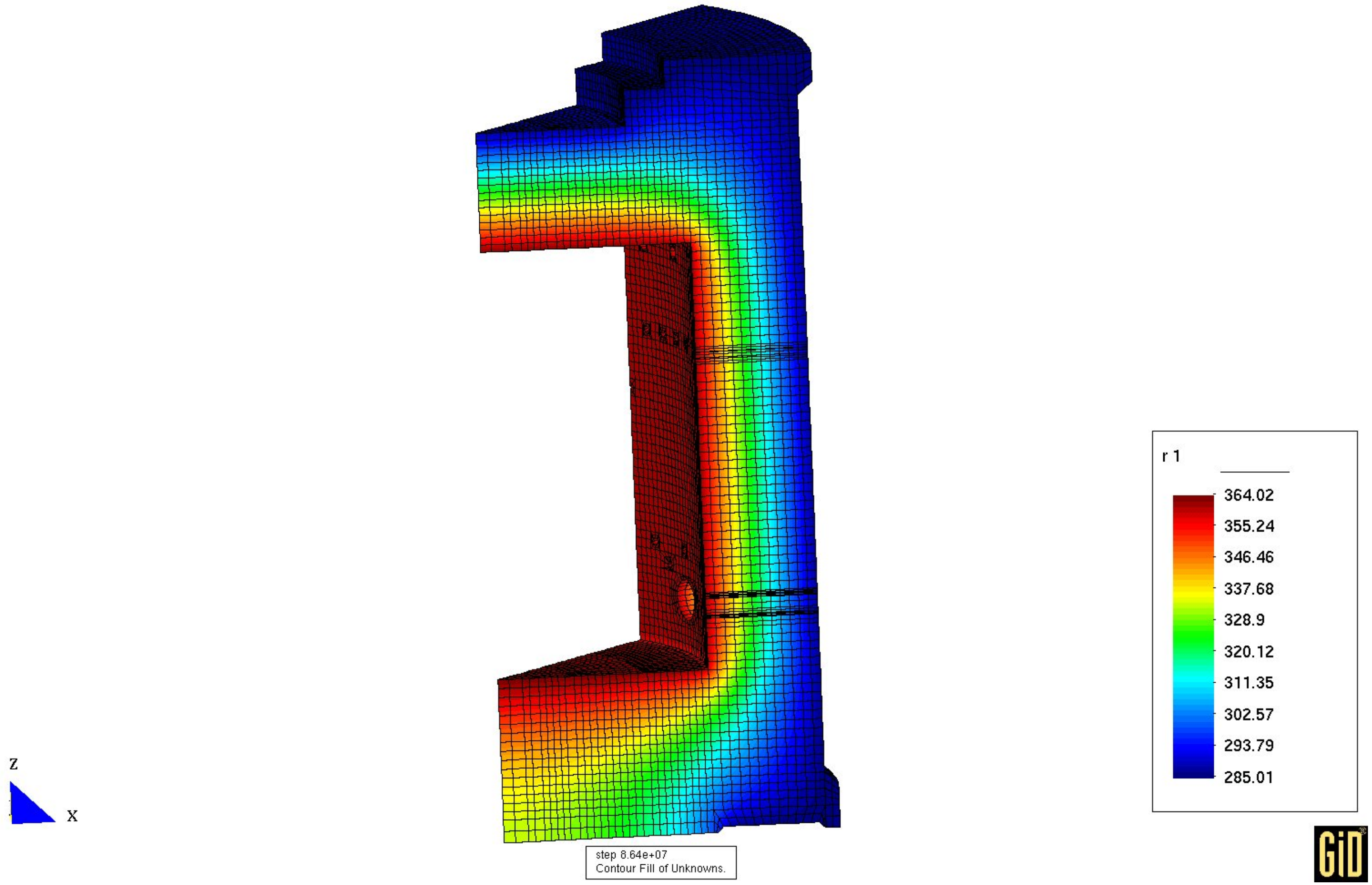

z
x
step 3.456e+06
Contour Fill of Unknowns.
r 1
360.79
352.37
343.95
335.53
327.11
318.69
310.26
301.84
293.42
285


z
x
step 4.752e+06
Contour Fill of Unknowns.
r 1
361.41
352.92
344.43
335.94
327.45
318.96
310.47
301.98
293.49
285

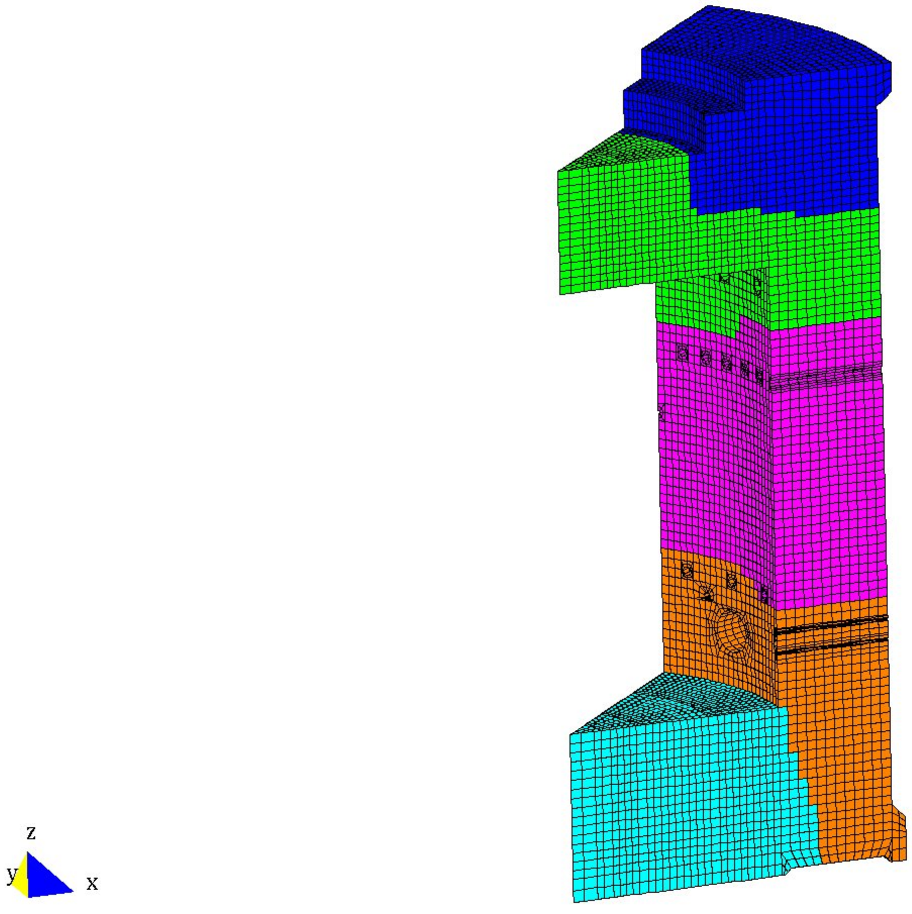

r 1
362.19
353.61
345.04
336.46
327.88
319.31
310.73
302.15
293.58
285
step 7.776e+06
Contour Fill of Unknowns.
Z
X

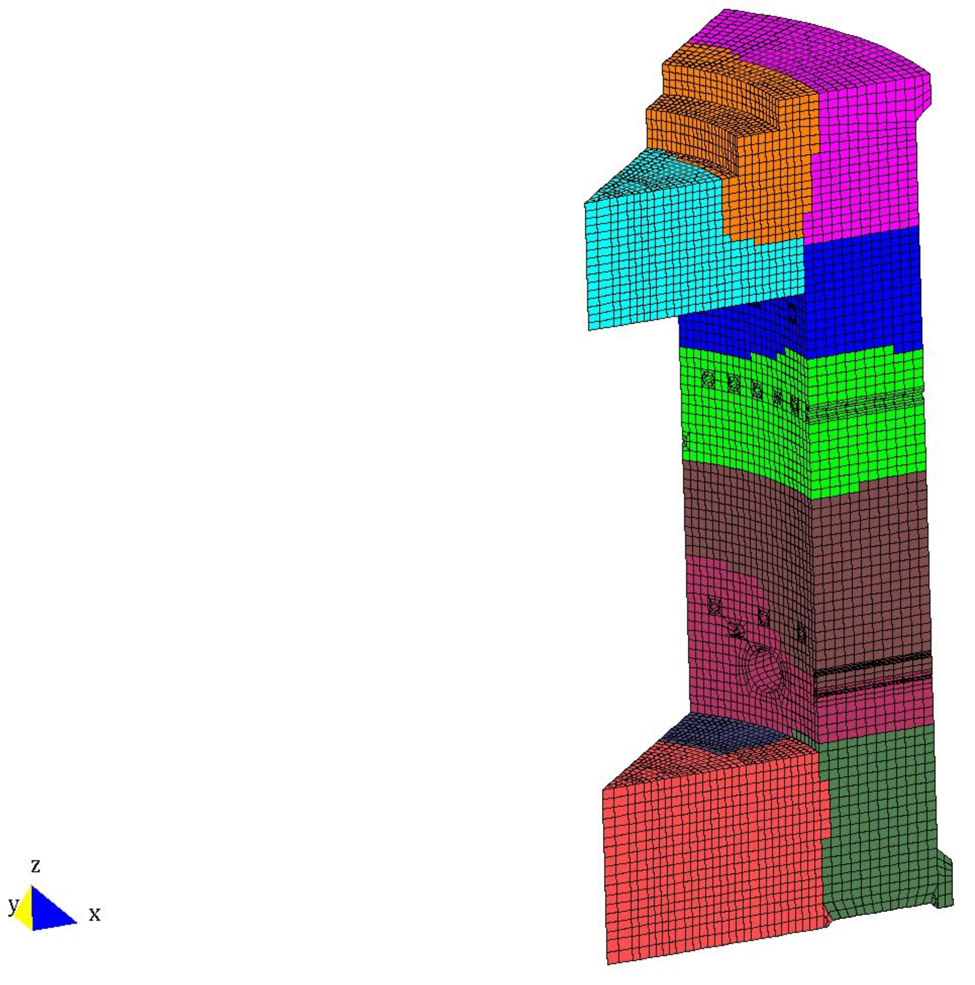

r 1
362.81
354.17
345.52
336.88
328.23
319.59
310.94
302.3
293.65
285.01
z
x
step 1.296e+07
Contour Fill of Unknowns.

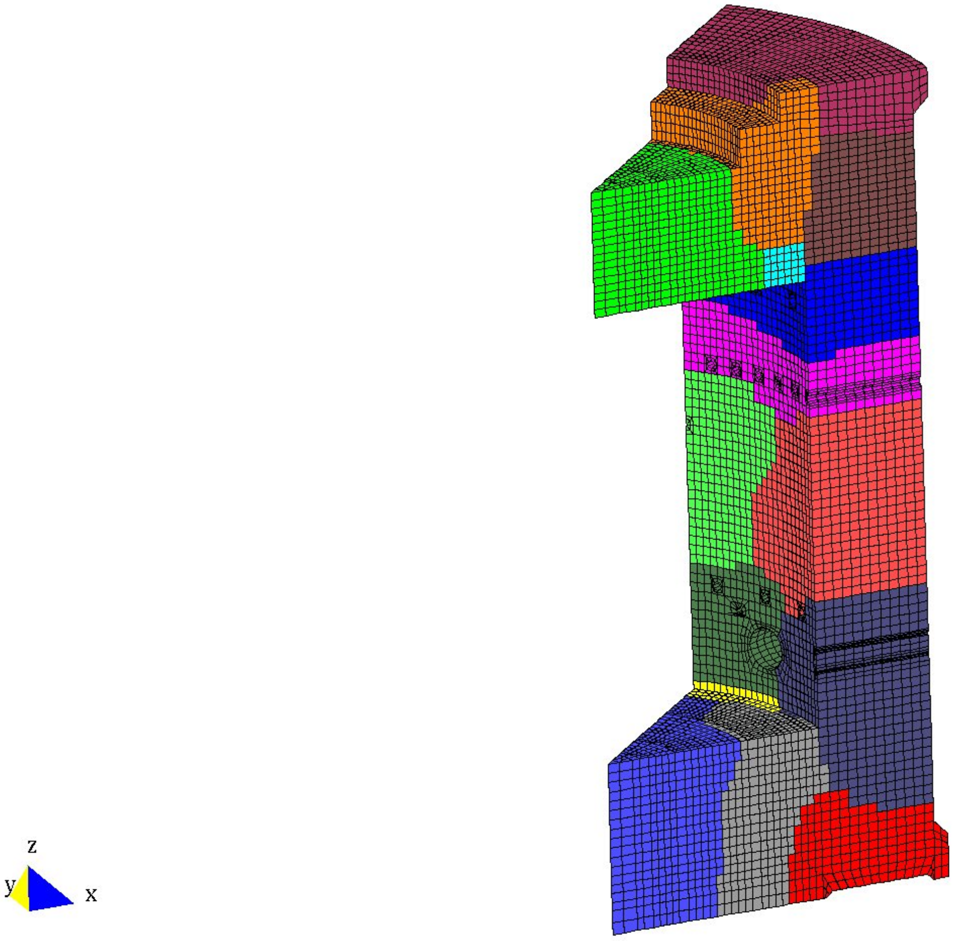

z
x
step 2.2464e+07
Contour Fill of Unknowns.
r 1
363.36
354.65
345.95
337.24
328.53
319.83
311.12
302.42
293.71
285.01

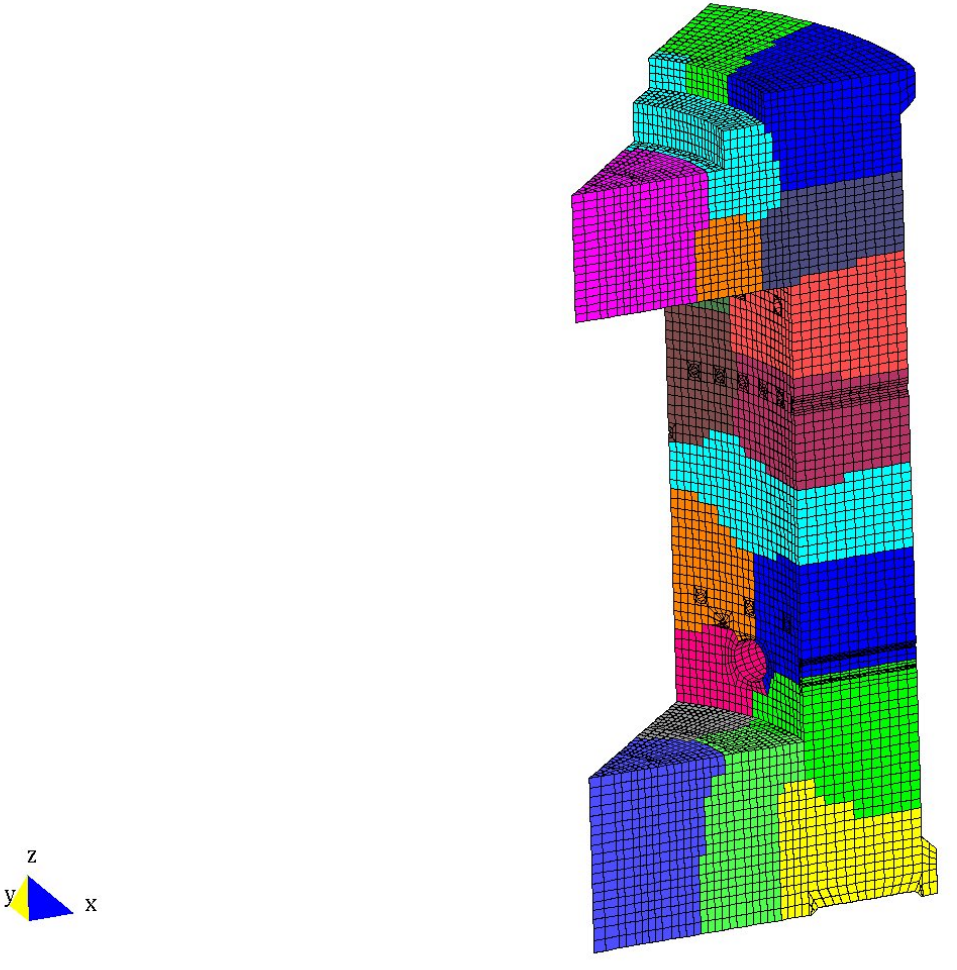

z
x
step 8.64e+07
Contour Fill of Unknowns.
r 1
364.02
355.24
346.46
337.68
328.9
320.12
311.35
302.57
293.79
285.01

Multi-Processor Computing




z
y
x


z
y
x


z
y
x

NS	min NN	max NN	min NE	max NE
5	10,075	11,088	8,669	9,135
6	8,538	9,311	7,204	7,636
10	5,186	5,709	4,319	4,583
15	3,588	3,996	2,879	3,052
20	2,757	2,989	2,159	2,291

MECHANICS - SCHUR METHOD

NS	SKYLINE		SPARSE DIRECT	
	min NEM	max NEM	BFACT	AFACT
5	46,067,479	123,580,086		
6	36,487,896	92,439,891	939,944	31,170,789
10	16,501,854	47,168,007	525,801	15,861,789
15	10,259,294	26,925,436	372,697	11,974,662
20	5,912,943	15,750,221	294,416	8,090,730

MECHANICS - SCHUR METHOD

NS	SKYLINE			SPARSE DIRECT			NI
	FACT	CP	TOT	FACT	CP	TOT	
5	672	2	682	91	2	99	68
6	447	1	455	70	1	77	79
10	191	2	199	45	3	53	98
15	86	3	95	20	3	28	128
20	77	4	88	11	3	21	141

HEAT - SCHUR METHOD

NS	SKYLINE		SPARSE DIRECT	
	min NEM	max NEM	BFACT	AFACT
5	5,906,791	14,471,303		
6	4,758,609	10,832,412	110,991	3,462,787
10	2,230,082	5,526,960	68,178	2,341,530
15	1,327,305	3,121,606	43,860	1,330,333
20	794,993	1,825,739	33,374	1,019,792

HEAT - SCHUR METHOD

NS	SKYLINE			SPARSE DIRECT			NI
	FACT	CP	TOT	FACT	CP	TOT	
5	30	0	32	28	0	30	8
6	19	0	21	22	0	23	10
10	8	0	9	15	0	16	8
15	3	0	4	6	0	7	10
20	3	0	4	4	0	5	9

MECHANICS - FETI-DP METHOD

NS	SKYLINE		SPARSE DIRECT	
	min NEM	max NEM	BFACT	AFACT
6	33,388,781	50,528,530	939,944	22,196,853
10	11,699,158	23,266,990	568,096	9,534,078
15	6,364,124	15,221,839	393,936	5,122,881
20	3,926,395	7,348,103	297,641	4,183,227

MECHANICS - FETI-DP METHOD

NS	SKYLINE			SPARSE DIRECT			NI
	FACT	CP	TOT	FACT	CP	TOT	
6	143	16	181	44	17	87	24
10	54	15	80	12	13	38	45
15	32	12	54	5	9	24	54
20	10	6	21	4	6	16	54

HEAT - FETI-DP METHOD

NS	SKYLINE		SPARSE DIRECT	
	min NEM	max NEM	BFACT	AFACT
6	4,364,303	5,954,933	114,227	2,240,062
10	1,593,704	2,716,834	64,641	1,137,598
15	813,501	1,768,244	44,153	559,446
20	510,241	853,407	35,035	464,611

HEAT - FETI-DP METHOD

NS	SKYLINE			SPARSE DIRECT			NI
	FACT	CP	TOT	FACT	CP	TOT	
6	7	1	8	14	4	18	12
10	2	1	3	4	1	6	15
15	2	0	3	2	1	3	16
20	0	1	2	1	1	2	16

COMPARISON

	1		6				20			
			SCHUR		FETI-DP		SCHUR		FETI-DP	
	CG	SD	SKY	SD	SKY	SD	SKY	SD	SKY	SD
M	57.7	338.4	455	77	181	87	88	21	21	16
T	1.5	137.5	21	23	8	18	4	5	2	2

Conclusions

- domain decomposition methods are useful for simulations of real problems
- domain decomposition methods speed up computation
- domain decomposition methods enable more complicated models and finer meshes