On the incompressible fluid flow in a journal bearing, the viscosity depending on pressure

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Abstract

The intention of this work is to present, in the perspective of numerical results, one of the recent generalisations of the Navier-Stokes model of fluid motion. The main aim is to follow the theoretical results achieved in [4] and to study the capabilities of the constitutive model, for which we can prove theoreticaly that there exists a solution.

Governing equations

An *homogeneous incompressible* lubricant fills the domain. The *steady motion* is described by the following equations (density ρ is a constant):

$$\operatorname{div} \boldsymbol{v} = 0 \quad \text{in } \Omega \tag{1}$$

$$\rho[\nabla \boldsymbol{v}]\boldsymbol{v} = \operatorname{div} \boldsymbol{T} \quad \text{in } \Omega. \tag{2}$$

The bearing wall is held fixed, while the journal rotates along its own axis; the *no-slip* Dirichlet boundary conditions complete the system (1)-(2):

$$\boldsymbol{v}_0 = \boldsymbol{0} \quad \text{on } \Gamma_B \subset \partial\Omega \text{ (the outer circle)}, \tag{3}$$

$$\boldsymbol{v}_0 = v_0 \boldsymbol{\tau} \quad \text{on } \Gamma_J \subset \partial\Omega \text{ (the inner circle)}, \tag{4}$$

where $v_0 = \omega R_J$ is given. We fix the pressure by setting the condition

$$\frac{1}{|\Omega_0|}\int_{\Omega_0} p \,\mathrm{d}x = p_0, \qquad \Omega_0 \subset \Omega. \tag{5}$$

The Cauchy stress tensor $\boldsymbol{T}$ we consider in the form

$$\boldsymbol{T} = -p\boldsymbol{I} + 2\nu(p, |\boldsymbol{D}|^2)\boldsymbol{D}, \tag{6}$$

where $|\boldsymbol{D}|^2 = \operatorname{tr} \boldsymbol{D}^2$, and $\boldsymbol{D} = \frac{1}{2}(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T)$. For a Newtonian fluid, one sets a constant viscosity

$$\nu = \nu_0, \tag{7}$$

leading to Navier-Stokes equations. When dealing with huge pressures, the viscosity should depend on the pressure. Based on experiments, the following *pressure-thickening* model is often used:

$$\nu = \nu_0 \mathrm{e}^{\alpha p}. \tag{8}$$

Based on our theoretical results, we propose a model which is, withal, *shear-thinning*

$$\nu = \nu_0(\nu_1 + \mathrm{e}^{-q\alpha\bar{p}} + \nu_2|\boldsymbol{D}|^2)^{\frac{r-2}{2}}. \tag{9}$$

Here $\bar{p} = \max\{p, p_-\}$, with p_- a given constant.

Existence of the solution

The mathematical results concerning the fluids with pressure dependent viscosities are rare; specially, there is no existence theory applicable to our problem when using the model (8).

There are recent results which includes the model (9). In [4] the existence of a weak solution to the system (1)-(5) is established. These results strongly use the *shear-thinning* properties of the model. For details see [2, 6].

Journal bearings

In their simplest form, a journal and its bearing consist of two eccentric, rigid, cylinders. The bearing is held stationary while the journal rotates at an angular velocity ω.

A continuous fluid film separates the solid surfaces. The film is generated and maintained by the viscous drag of the surfaces as they are sliding relative to one another.

As an approximation, we reduce the motion to the plane perpendicular to the axial direction, considering the 2D geometry in Figure 1.

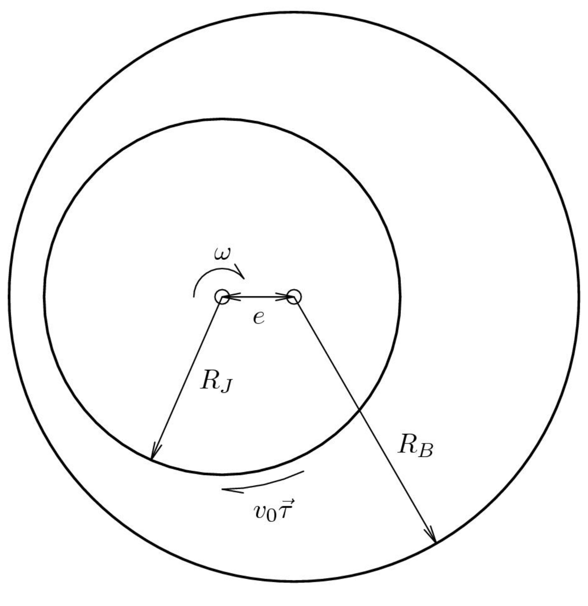

Figure 1: *Journal bearing geometry*

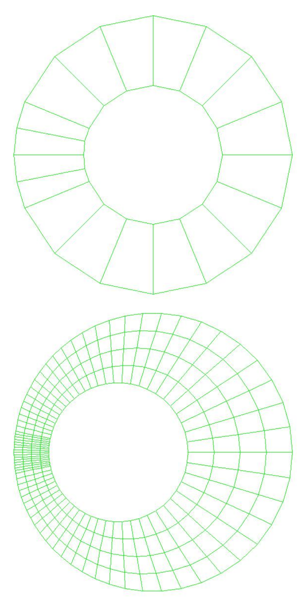

Figure 2: *FEM meshes*

The domain is an eccentric annular ring, the radii of circles being R_B and R_J and the distance between their centres being e. Defining the radial clearance $C = R_B - R_J$ we denote $\varepsilon = e/C$, $\varepsilon \in \langle 0, 1\rangle$ the eccentricity ratio.

Discretisation

We use standard Galerkin FEM, with quadrilateral mesh (velocities being in Q_2 and pressures in P_1 on each element). The system of nonlinear algebraic equations is solved by Newton iterations with damping, the Jacobian matrix being approximated by finite differences and the application of its inverse being performed by `UMFPACK` solver.

The discretisation is made on concentric annulus with $R_J = 0.5$ and $R_B = 1.0$, and the equations are transformed to obtain solution for given parameters R_B, R_J and ε, see Figure 2.

Our setting is:

$$R_B = 31.29\,\mathrm{mm}, \quad R_J = 31.25\,\mathrm{mm}, \quad \omega = 250\,\mathrm{rad/s}$$

$$\rho = 820\,\mathrm{kg/m^3}, \quad \nu_0 = 0.05\,\mathrm{Pa.s}$$

In the numerical experiments below, we confront the model (9) with the model (8). See also [3, 5] for a similar study.

Numerical experiments

First we examine the exponential model (8), with different values of parameter α, see Figure 3:

$$\alpha = 0.0, \quad \alpha = 10^{-7}, \quad \alpha = 2\times 10^{-7}$$

(See [1] for values determined by experiments.)

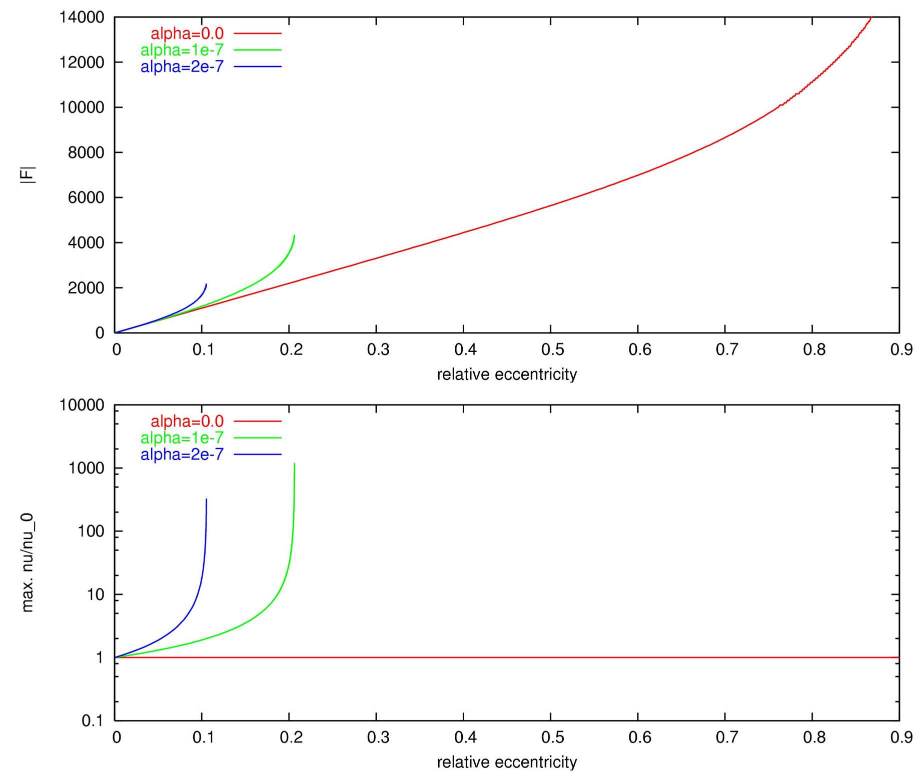

Figure 3: *Reaction force and max. viscosities for model* (8)

The graphs for $\alpha > 0$ cease due to the fact, that we were no more able to find a numerical solution for higher ε.

For $\alpha = 2\times 10^{-7}$ we compare the previous results with the model (9), where we set

$$r = 3/2 \text{ and } q = 4 \qquad \text{(it holds } (\mathrm{e}^{-q\alpha\bar{p}})^{\frac{r-2}{2}} = \mathrm{e}^{\alpha\bar{p}})$$

and where

$$\nu_1 = 10^{-10} \qquad \nu_2 = 10^{-8}, \quad \nu_2 = 10^{-10}, \quad \nu_2 = 10^{-12}$$

see the magnitudes of reacting forces on Figure 4.

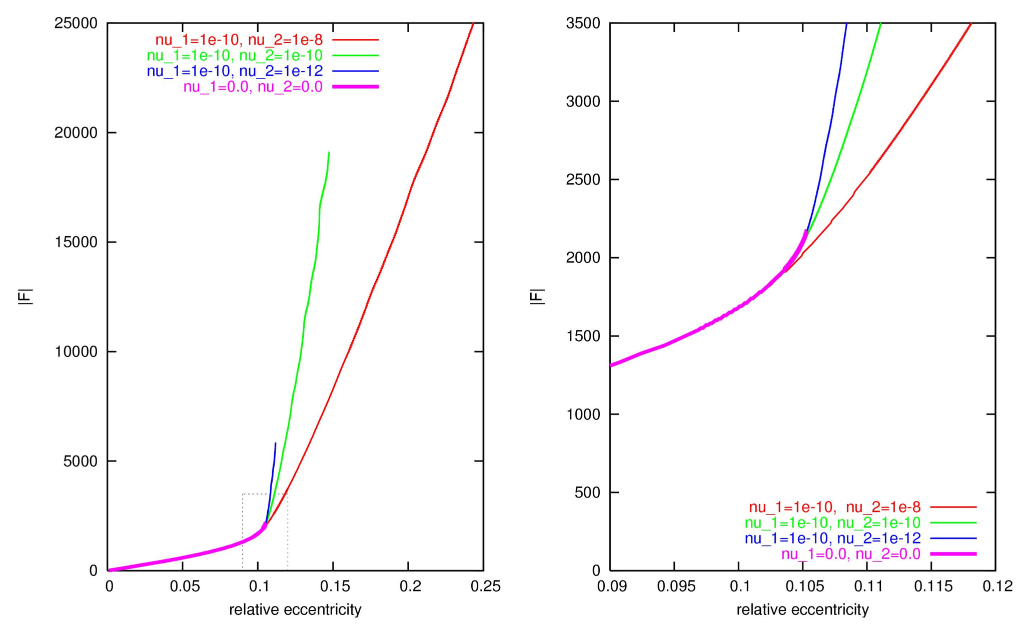

Figure 4: *Reacting force magnitude for model* (8)

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