On the Modeling of Entropy Producing Processes

K. R. Rajagopal

Texas A&M University

August 2007

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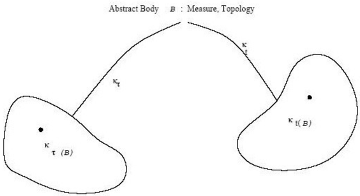

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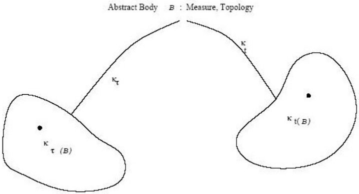

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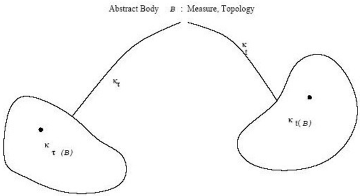

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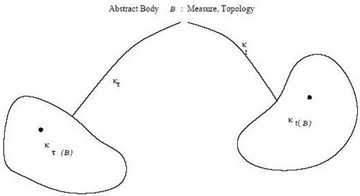

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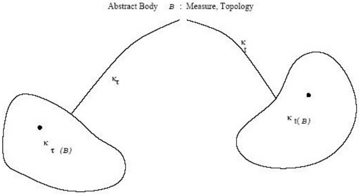

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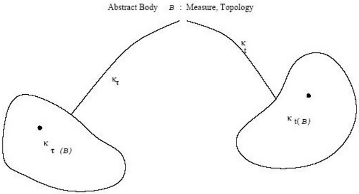

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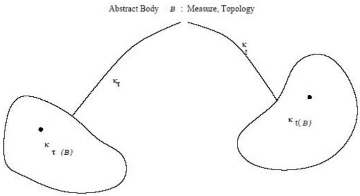

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Relative Motion

$$\xi = \chi_{\kappa_R}\left(\chi_{\kappa_R}^{-1}(\mathbf{x}, t), \tau\right) = \chi_t(\mathbf{x}, \tau). \tag{2}$$

Kinematics

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$$\phi = \tilde{\phi}(\mathbf{x}, t); \qquad \mathrm{grad}\phi := \frac{\partial\tilde{\phi}}{\partial\mathbf{x}}; \qquad \frac{\partial\phi}{\partial t} := \frac{\partial\tilde{\phi}}{\partial t} \tag{6}$$

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$$\mathbf{T} = \begin{cases} -p_1\mathbf{I} + 2\mu_1\mathbf{D}_{\kappa_{(t)}} & \forall t \leq t', \\ -p_2\mathbf{I} + \hat{\mu}_1\mathbf{D}_{\kappa_{(t)}} + \hat{\mu}_2(\mathbf{D}_{\kappa_{(t)}})^2 & \forall t > t' \end{cases} \tag{12}$$

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- Natural configuration $\approx$ Equivalence class of configurations.

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- Different natural configurations are accessed during different processes. The natural configuration is a part of the specification of the "state" of the body.

Natural Configurations

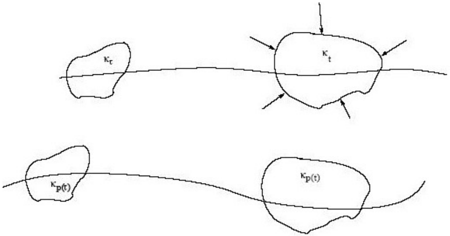

Figure: $\kappa_{\mathrm{p}(\tau)}$ Natural configuration corresponding to κ_τ and $\kappa_{\mathrm{p(t)}}$ natural configuration corresponding to κ_{t}

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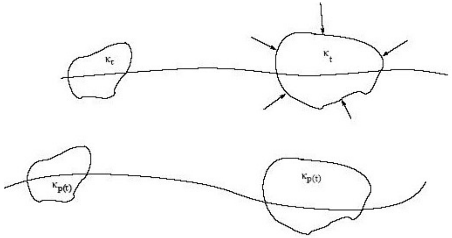

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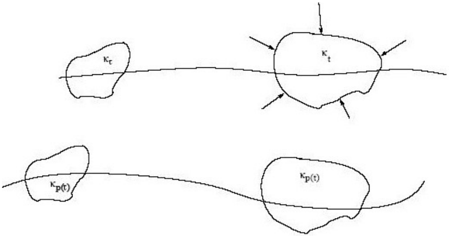

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- If one inhomogeneously deforms a body and then removes the traction, it is possible that the unloaded body will not fit together compatably and be simultaneously stress free in an Euclidean space.

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- Henceforth, for the sake of illustration, let us assume homogeneous deformations.

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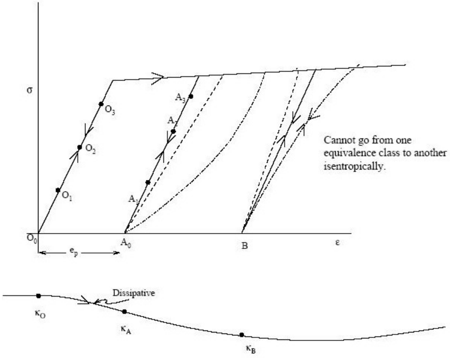


Figure: Traditional Plasticity

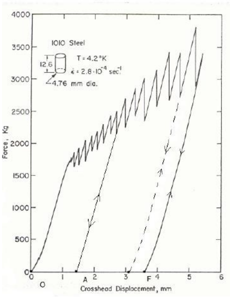
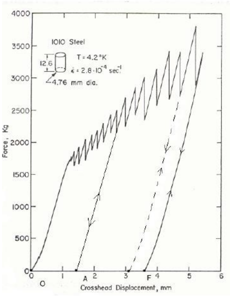


Figure: Modulo variants, we have two natural configurations, that corresponding to O and F, and these two natural configurations have different material symmetries.

Twinning



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- In twinning there are a finite number. As many as the number of variants.

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- Classical theories are trivial examples:
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  - In classical fluids the current configuration is the natural configuration.

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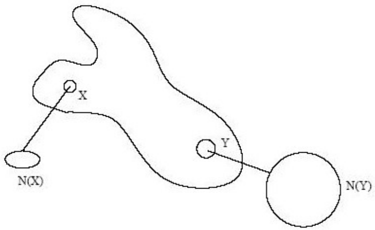

Figure: Configuration as a local notion

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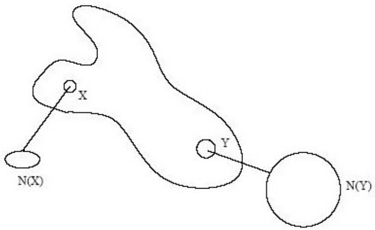

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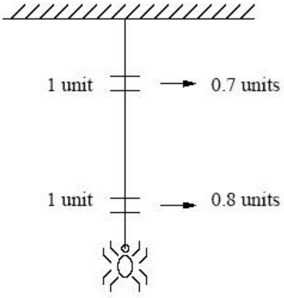

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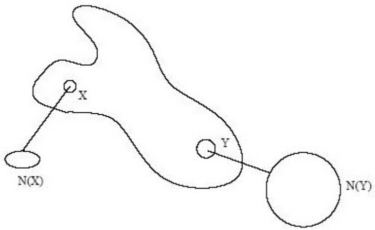

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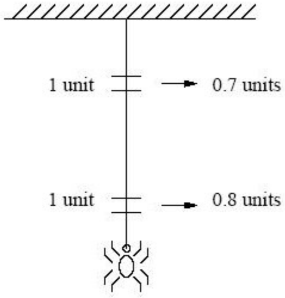

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- New material is laid in a stressed state. It can have a different natural configuration than the material laid down previously.

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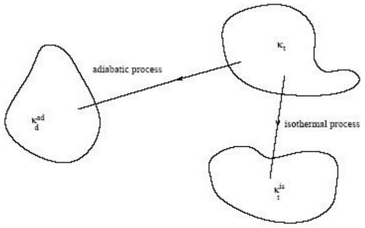

Figure: Non-uniqueness of stress-free state (Modulo rigid motion)

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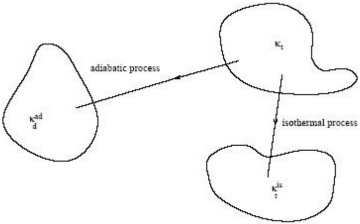

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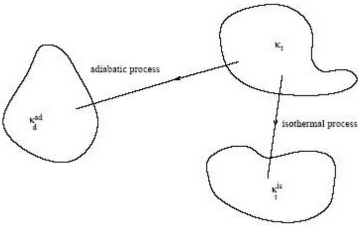

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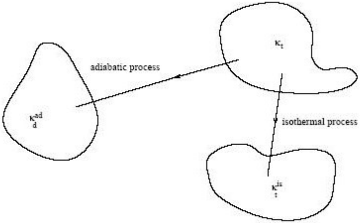

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- More than one Natural Configuration can be associated with the current deformed configuration.

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Balance of Angular Momentum

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- Here $\mathbf{T}$ = Stress, η= Specific entropy, θ= Temperature, $\mathbf{q}$= Heat flux vector, r= Radiant heating

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- The maximization of entropy production makes choices amongst possible response functions. For instance, it will pick a rate of dissipation (or entropy production) from amongst a class of candidates.

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- There is no contradiction between these two criteria:

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- For a class of materials, such a choice leads to a Liapunov function that decreases with time to a minimum value (Onsager/Prigogine- Minimum entropy production criterion). Rajagopal and Srinivasa(2003), Proc. Royal Society.

- There is no contradiction between these two criteria:
  - Maximization of entropy production to pick constitutive equations and the minimization of entropy production with time once a choice has been made. (Rajagopal and Srinivasa (2002)).

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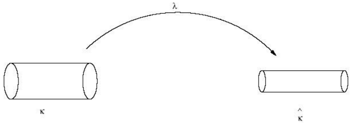
- For example one could have the crystalline material being orthotropic with the axis of othotropy being determined by the eigen-vectors of the stretch tensor or the symmetric part of the velocity gradient.

Symmetry Issues



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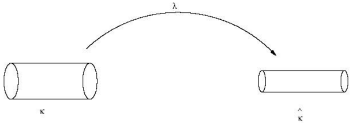

Classical Elasticity

$$\mathcal{G}_\kappa := \{\mathbf{H} \in \mathcal{U} | \mathbf{f}_\kappa(\mathbf{FH}) = \mathbf{f}_\kappa(\mathbf{F})\}, \tag{19}$$

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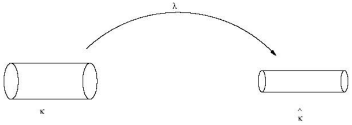

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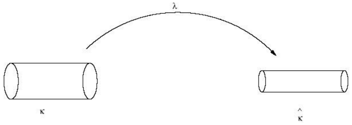

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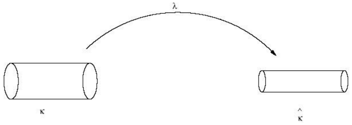

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- The symmetry group for a simple fluid is the unimodular group, $\mathcal{U}$ (Noll).

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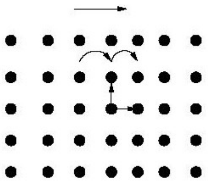

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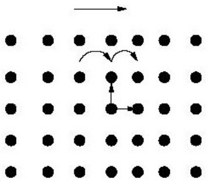

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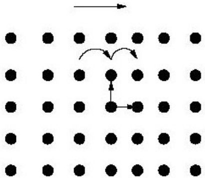

- After shearing the lattice structure remains the same.
- Contradicts Nolls rule.

Solidification and Melting

- "The properties of normal liquids are strictly isotropic; they possess no crystalline structure which singles out any one direction as different from another, while true solids (excluding glasses and amorphous phases) possess non-spherical symmetries which are characteristic of the regular arrangement of their molecules in a crystalline lattice. In order to go from a liquid to a crystalline phase, therefore, it is necessary to make a change of symmetry".

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  Pippard (1957)

- "Every transition from a crystal to a liquid and a liquid to a crystal, or a crystal to another with different symmetry is associated with the appearance or disappearance of some element of symmetry, . . . it can appear or disappear only as a whole, and not gradually".

  Landau (1967)

Reduced Energy-Dissipation Equation

$$\mathbf{T} \cdot \mathbf{L} - \rho\dot{\epsilon} + \rho\theta\dot{\eta} - \frac{\mathbf{q} \cdot \mathrm{grad}\theta}{\theta} = \rho\theta\xi := h \geq 0 \qquad (22)$$

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$$\Phi := \zeta + \lambda_1(\zeta - \mathbf{T}\cdot\mathbf{D} + \rho\dot{\psi}) + \lambda_2(\mathrm{tr}\mathbf{D}) \tag{25}$$

Q: Starting from the assumption that the stress depends on the density and the velocity gradient, how does one arrive at the Classical Navier-Poisson-Stokes Fluid (compressible and incompressible)?

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Isotropy

$$\mathbf{Q}\hat{\mathbf{f}}(\rho, \mathbf{D})\mathbf{Q}^{\mathrm{T}} = \hat{\mathbf{f}}(\rho\mathbf{Q}\mathbf{D}\mathbf{Q}^{\mathrm{T}}), \ \forall \mathbf{Q} \in \mathbf{\Theta} \tag{28}$$

Representation Theorem

$$\hat{\mathbf{f}}(\rho, \mathbf{D}) = \alpha_1 \mathbf{I} + \alpha_2 \mathbf{D} + \alpha_2 \mathbf{D}^2 \tag{29}$$

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Incompressibility

$$\mathbf{T} = \hat{\mathbf{f}}(\mathbf{D}) = -p\mathbf{I} + 2\mu\mathbf{D} \tag{31}$$

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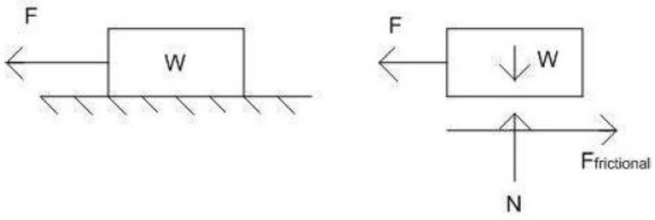
- Density changes in liquids in certain applications(wherein the pressure (normal stresses) changes by several orders of magnitude) are of the order of a few percent, while the viscosity changes by factor of 10^7 to 10^8 !!!

- Elastohydrodynamic Lubrication, Szeri (1998)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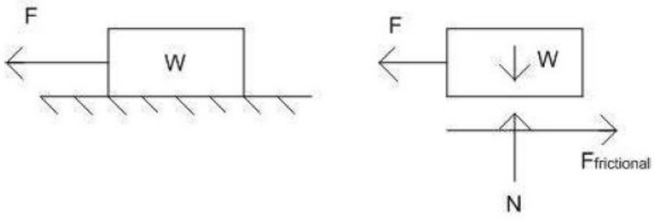
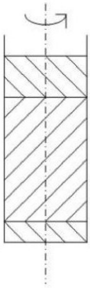
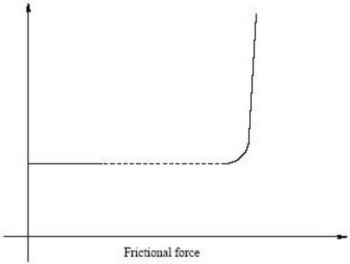


- Frictional force definitely depends on the normal force for solids. Why should it be any different for fluids?

- Coulombs erroneous conclusions on the basis of his experiments:





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  Let us now consider in what cases it is allowable to suppose μ to be independent of the pressure. It has been concluded by Du Buat from his experiments on the motion of water in pipes and canals, that the total retardation of the velocity due to friction is not increased by increasing the pressure... I shall therefore suppose that for water, and by analogy for other incompressible fluids, μ is independent of the pressure . . .

Barus (1891)

$$\mu = A \exp(\alpha p), \quad \alpha \ - \ \text{constant}, \ \ \alpha \geq 0 \tag{32}$$

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A: It is not correct to make such an assumption. Moreover, it depends on what one means by the constraint response.

- The following representation stems from assuming that the constraint response does no work, in this case the constraint being $\mathrm{tr}\mathbf{D} = \mathbf{0}$;

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  We now restrict ourselves to systems for which the net virtual work of forces of constraint is zero. We have seen that this condition holds for rigid bodies and it is valid for a large number of other constraints. Thus, if a particle is constrained to move on a surface, the force of constraint is perpendicular to the surface, while the virtual displacement must be tangent to it, and hence the virtual work vanishes. This is no longer true if sliding friction forces are present, and **we must exclude such systems from our formulation**.

- **Gauss** (1829 - Translated into English and published in the Philosophical Magazine in 1841):

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 The motion of a system of material points connected together in any manner whatsoever, whose motions are modified by any external restraints whatsoever, proceeds in every instance in the greatest possible accordance with free motion, or under the least possible constraint; *the measure of the constraint which the whole system suffers in every particle of time being considered equal to the sum of products of the square of the deviation of every point from its free motion into its mass.*

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$$\mathbf{f}(\mathbf{T}, \mathbf{D}) = \mathbf{0} \qquad (37)$$

Thank You