Solving Quantified Constraints with RSolver

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The Problem

Given: expression with

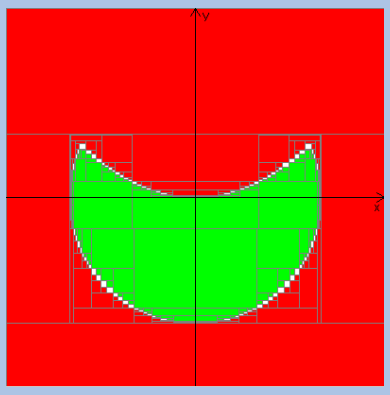
- Quantifiers: $\forall$, $\exists$
- Boolean connectives: $\vee$, $\wedge$, $\neg$
- Predicates Symbols: $=$, $\leq$, $<$
- Function Symbols: Rational constants, $+$, $\times$, $\sin$, $\exp$, $\ldots$

Find: Description of set of solutions.

Restriction: all variables bounded

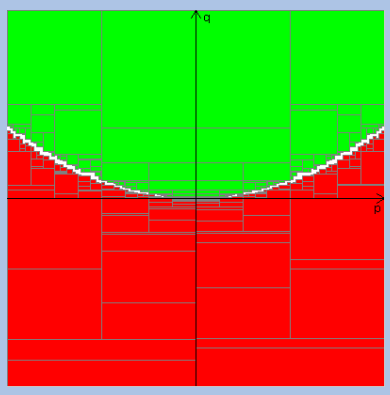
Example 1

$$x^2 + y^2 - 1 \leq 0 \wedge 2y \leq x^2$$



Example 2

$$\forall x \in [[-100, 100]]\ x^2 + px + q \geq 0$$



The Method

- deduce information using interval/constraint propagation techniques
- – split intervals,
  – rewrite $\forall x \in [\underline{a}, \overline{a}]\phi$ to $\forall x \in [\underline{a}, a^*]\phi \wedge \forall x \in [a^*, \overline{a}]\phi$,
  – rewrite $\exists x \in [\underline{a}, \overline{a}]\phi$ to $\exists x \in [\underline{a}, a^*]\phi \vee \exists x \in [a^*, \overline{a}]\phi$
- terminates for all well-posed cases ($f = 0$ interpreted as $f \leq 0 \wedge f \geq 0$)

`http://rsolver.sourceforge.net`

RSolver