Non-monotonic Reasoning with Various Kinds of Preferences in the Relational Data Model Framework

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Abstract. *The paper gives an overview of recent advances in the field of logic of preference and discusses their applicability in the frame of the relational data model. Namely, non-monotonic reasoning mechanisms with various kinds of preferences are reviewed in detail, and a way of suiting them to practical database applications is presented. These mechanisms enable to reason simultaneously about sixteen strict and non-strict kinds of preferences, including ceteris paribus preferences. To make the mechanisms useful for practical applications, the assumption of preference specification consistency has to be loosened. This is achieved in two steps: firstly, all the preference specifications are generalized to permit uncertainty, and secondly, not a total pre-order on worlds but a partial pre-order on worlds is used in the semantics, which enables to indicate some kind of conflict among worlds by their incomparability. Most importantly, the semantics of set of preferences is related to that of a disjunctive logic program.*

1 Introduction

All to often no reasonable answer is returned by an SQL-based search engine though one has tried hard writing query to match one's personal preferences closely. The case of repeatedly receiving *empty query result* is extremely disappointing to the user. On the other hand, leaving out some conditions in the query often leads to another unpleasant extreme: an *overloading* with lots of mostly *irrelevant information*.

This observation stems from the fact that traditional database query languages treat all the requirements on the data as mandatory, hard ones. However, it is natural to express queries in terms of both hard as well as soft requirements, i.e., preferences, in many applications. In the "real world", preferences are understood in the sense of wishes: in case they are not satisfied, database users are usually prepared to accept worse alternatives. Thus preferences require a paradigm shift from exact matches towards a best possible matchmaking.

The paper presents a work in progress aiming at **simultaneous usage of various preferences (including set preferences) with general, preference logic based semantics in the context of database queries**. The objective is to provide database users with a language that is declarative, can be used to define such database queries that not necessarily all answers but rather the best, the most preferred ones are returned, includes various kinds of preferences, and has an intuitive, well defined semantics allowing for conflicting preferences.

In section 2, the basic concepts of logic of preference and non-monotonic logic of preference are defined. In section 3, basic concepts and key features of the proposed approach are presented, and section 4 concludes the paper.

2 Preliminaries

The logic of preference has been studied since the sixties as a branch of philosophical logic: Logicians and philosophers have been attempting to define the one well-formed logic that people should follow when expressing preferences.

2.1 Logic of preference

It is Von Wright's essay [10] that tries to give the first axiomatization of a logic of preference. The general idea is that the expression "a is preferred to b" should be understood as the preference of a state (a world) where a occurs over a state where b occurs. Von Wright expressed a theory based on five axioms. The problem is that empirical observation of human behavior provides counterexamples of this axiomatization.

Later, Von Wright [11] introduced a more general frame to define preferences, updating also the notion of ceteris paribus preferences. In this approach, he considers a set S of n logically independent states of affairs and the set $W = \mathcal{P}(S)$ of 2^n combinations of the elements of S. An s-world is called any element of W that holds when s holds. In the same way is defined a C_i-world, where C_i is a combination of elements of S. Now, von Wright gives two definitions (strong and weak) of "s is preferred to t *under the circumstances* C_i":

1. (strong): s is preferred to t under the circumstances C_i iff every C_i-world that is also an s-world and not a t-world is preferred to every C_i-world that is also a t-world and not an s-world.

2. (weak): s is preferred to t under the circumstances C_i iff some C_i-world that is also an s-world is preferred to some C_i-world that is also a t-world, and no C_i-world which is a t-world is preferred to any C_i-world which is an s-world.

Finally, if s is preferred to t under all circumstances C_i, according to either definition, then s is said to be preferred to t ceteris paribus.

It can be concluded that the philosophical discussion about preferences failed the objective to give a unifying frame of generalized preference relations that could hold for any kind of states, based on well-defined axiomatization.

More recently, Von Wright's ideas and the discussion about "logical representation of preferences" attracted attention again. For instance Doyle and Wellman [3] give a modern treatment of preferences ceteris paribus. On the other hand, Boutilier [2] pioneers a new way of looking at preference logic by augmenting a basic modal language. His work is the base of the recent work of van Benthem, Otterloo and Roy [9], who reduce preference logic to a basic (multi)modal language augmented with tho so-called *existential modality*. Their semantics does not include ceteris paribus property of preferences.[1]

2.2 Logic of preferences

A drawback of the present state of the art in the logic of preference is that proposed logics typically formalize only preference of one kind. Consequently, when formalizing preferences, one has to choose which kind of preference statements are used for all preferences under consideration.

To study the interaction among kinds of preferences, a non-monotonic preference logic for various kinds of preferences, *logic of preferences* – in contrast to the usual reference to the *logic of preference*, has been recently developed by Kaci and Torre [5, 6]. They have developed algorithms for a non-monotonic preference logic for sixteen kinds of preferences: four basic types, each of them strict or non-strict, with or without ceteris paribus proviso.

To describe ceteris paribus preference, a general construction proposed by Doyle and Wellman [3] is employed. Their language for preference built over a set of propositions is defined inductively from propositional variables. They mean by *proposition* a set of individual objects, elements of a set W. These individual objects can be understood as worlds, i.e., truth assignments for propositional variables. In other words, a propositional formula is identified with worlds – fulfilling truth assignments, and the powerset $\mathcal{P}(W)$ is taken to be the set of all propositional formulas.

Their ceteris paribus preferences are based on a notion of contextual equivalence:

Definition 1. ***(Contextual equivalence)****[3, Def.4] Let W be a set of worlds and $\xi(W)$ be the set of equivalence relations on W. A contextual equivalence on W is a function $\eta : \mathcal{P}(\mathcal{P}(W)) \to \xi(W)$ assigning to each set of propositional formulas $\{\varphi, \psi, \ldots\}$ equivalence relation $\eta(\varphi, \psi, \ldots)$.*

If $w\ \eta(\varphi, \psi, \ldots)\ w'$, we usually write

$$w \equiv w' \mod{}_{\eta}(\varphi, \psi, \ldots) \ .$$

Definition 2. ***(Preference model)*** *A preference model $\mathcal{M} = \langle W, \succeq, \eta \rangle$ is a triplet in which W is a set of worlds, $\succeq$ is a total pre-order, i.e., a relation which is complete, reflexive, and transitive, over W, and η is a contextual equivalence function on W.*

Definition 3. ***(Comparative greatness)****[3, Def.5] We say that "φ is weekly greater than ψ," written $\varphi \trianglerighteq \psi$, is satisfied in the model $\mathcal{M}$, written $\mathcal{M} \models \varphi \trianglerighteq \psi$, iff $w_1 \succeq w_2$ whenever*

1. $w_1 \models \varphi \wedge \neg\psi$,
2. $w_2 \models \neg\varphi \wedge \psi$, *and*
3. $w_1 \equiv w_2 \mod{}_{\eta}(\varphi \wedge \neg\psi, \neg\varphi \wedge \psi)$.

This definition of ceteris paribus preferences seems very close to the intended semantics behind von Wright's principles. Preferences of φ over ψ are defined as preferences of $\varphi \wedge \neg\psi$ over $\neg\varphi \wedge \psi$, which is standard and known as von Wright's expansion principle [10]. Also, note that if the equivalence relation $\eta(\varphi \wedge \neg\psi, \neg\varphi \wedge \psi)$ is the universal relation, i.e., an equivalence relation with only one equivalence class, then the ceteris paribus preference reduces to strong condition (φ is preferred to ψ when each $\varphi \wedge \neg\psi$ is preferred to all $\neg\varphi \wedge \psi$).

The following proposition [1] shows that Def.3 reduces a preference with ceteris paribus proviso to a set of preferences for each equivalence class of the equivalence relation.

Proposition 1. *[1, Prop.1] Assume a finite set of propositional variables, and let $\epsilon(\eta, \varphi, \psi)$ be the set of propositional formulas which are true in all worlds of an equivalence class of $\eta(\varphi, \psi)$, but false in all others: $\{\chi | \exists w \forall w_2 (w_1 \equiv w_2 \mod{}_{\eta}\varphi, \psi \iff w_1 \models \chi)\}$. We have that "$\varphi$ is weakly greater than ψ" is satisfied in the model $\mathcal{M}$ iff for all propositions $c \in \epsilon(\eta, \varphi \wedge \neg\psi, \neg\varphi \wedge \psi)$, we have that $w_1 \succeq w_2$ whenever*

1. $w_1 \models \varphi \wedge \neg\psi \wedge c$,
2. $w_2 \models \neg\varphi \wedge \psi \wedge c$.

[1] For more detailed survey of the origin of preference logic in the work of von Wright refer to [4].

The logical language introduced in [6] extends propositional logic with sixteen kinds of preferences:

Definition 4. ***(Language)*** *[6, Def.3] Given a finite set of propositional variables $p, q, \ldots$, the set L_0 of propositional formulas and the set L of preference formulas is defined as follows:*

$L_0 \ni \varphi, \psi$: $p|(\varphi \wedge \psi)|\neg\varphi$

$L \ni \Phi, \Psi$: $\varphi \ {}^{x}{>}^{y} \ \psi | \varphi \ {}^{x}{\geq}^{y} \ \psi | \varphi \ {}^{x}{>}^{y}_{c} \ \psi | \varphi \ {}^{x}{\geq}^{y}_{c} \ \psi |$
$\neg\Phi|(\Phi \wedge \Psi)$ *for* $x, y \in \{m, M\}$

Definition 5. ***(Monotonic semantics)*** *[6, Def.4] Let $\mathcal{M}$ be a preference model. When $x = M$ we write $x(\varphi, \mathcal{M})$ for*

$$\max(\varphi, \mathcal{M}) = \{w \in W | w \models \varphi \wedge \forall w' \in W : w' \models \varphi \Rightarrow w \succeq w'\} \ ,$$

and analogously when $x = m$ we write $x(\varphi, \mathcal{M})$ for

$$\min(\varphi, \mathcal{M}) = \{w \in W | w \models \varphi \wedge \forall w' \in W : w' \models \varphi \Rightarrow w' \succeq w\} \ .$$

$\mathcal{M} \models \varphi \ {}^{x}{>}^{y} \ \psi$ *iff* $\forall w \in x(\varphi \wedge \neg\psi, \mathcal{M}),$
$\forall w' \in x(\neg\varphi \wedge \psi, \mathcal{M}) : w \succ w'$

$\mathcal{M} \models \varphi \ {}^{x}{\geq}^{y} \ \psi$ *iff* $\forall w \in x(\varphi \wedge \neg\psi, \mathcal{M}),$
$\forall w' \in x(\neg\varphi \wedge \psi, \mathcal{M}) : w \succeq w'$

$\mathcal{M} \models \varphi \ {}^{x}{>}^{y}_{c} \ \psi$ *iff* $\forall c \in \epsilon(\eta, \varphi \wedge \neg\psi, \neg\varphi \wedge \psi),$
$\forall w \in x(\varphi \wedge \neg\psi \wedge c, \mathcal{M}),$
$\forall w' \in x(\neg\varphi \wedge \psi \wedge c, \mathcal{M}) : w \succ w'$

$\mathcal{M} \models \varphi \ {}^{x}{\geq}^{y}_{c} \ \psi$ *iff* $\forall c \in \epsilon(\eta, \varphi \wedge \neg\psi, \neg\varphi \wedge \psi),$
$\forall w \in x(\varphi \wedge \neg\psi \wedge c, \mathcal{M}),$
$\forall w' \in x(\neg\varphi \wedge \psi \wedge c, \mathcal{M}) : w \succeq w'$

Moreover, logical notions are defined as usual, in particular:

$$S \models \Phi \iff \forall \mathcal{M} : \mathcal{M} \models S \Rightarrow \mathcal{M} \models \Phi \ .$$

Note that $\varphi \ {}^{m}{\geq}^{M}_{c} \ \psi$ is the Doyle and Wellmans's comparative greatness (Def.3).

In this paper, we are interested in a special kind of theories, namely preference specifications:

Definition 6. ***(Preference specification)*** *[6, Def.5] Let $\mathcal{P}_{\rhd}$ be a set of preferences of the form $\{\varphi_i \rhd \psi_i : i = 1, \ldots, n\}$. A preference specification $\mathcal{P}$ is a tuple $\langle \mathcal{P}_{\rhd} | \rhd \in \{ {}^{x}{>}^{y}, {}^{x}{\geq}^{y}, {}^{x}{>}^{y}_{c}, {}^{x}{\geq}^{y}_{c} \ | x, y \in \{m, M\}\}\rangle$, and $\mathcal{M}$ is its model iff it models all $\mathcal{P}_{\rhd}$:*

$$\mathcal{M} \models \mathcal{P}_{\rhd} \iff \forall(\varphi_i \rhd \psi_i) \in \mathcal{P}_{\rhd} : \mathcal{M} \models \varphi_i \rhd \psi_i$$

Corollary 1. *Observe that by Prop.1, we can replace ceteris paribus preferences, written ${}^{x}{>}^{y}_{c}$ or ${}^{x}{\geq}^{y}_{c}$, by sets of ordinary preferences without a ceteris paribus proviso. Consequently, we can restrict ourselves to the eight types of preferences without ceteris paribus clauses.*

2.3 Non-monotonic logic of preferences

Non-monotonic reasoning has been characterized by Shoham [8] as a mechanism that selects a subset of the models of a set of formulas, which we call distinguished models. Thus non-monotonic consequences of a logical theory are defined as all formulas which are true in the distinguished models of the theory.

An attractive property occurs when there is only one distinguished model, as then all non-monotonic consequences can be found by calculating the unique distinguished model and characterizing all formulas satisfied by this model. It has been proved in the literature that a unique distinguished model can be defined for the following sets of preferences: $\mathcal{P}_{\ {}^{m}{>}^{M}}$, $\mathcal{P}_{\ {}^{m}{>}^{m}}$, and $\mathcal{P}_{\ {}^{M}{>}^{M}}$.

Moreover, Kaci and Torre [6] have defined a distinguished model and proved ist uniqueness for

$$\langle \mathcal{P}_{\rhd} | \rhd \in \{ {}^{x}{>}^{y}, {}^{x}{\geq}^{y}, {}^{x}{>}^{y}_{c}, {}^{x}{\geq}^{y}_{c} \ | x \in \{m, M\}, y = M\}\rangle$$

and also for

$$\langle \mathcal{P}_{\rhd} | \rhd \in \{ {}^{x}{>}^{y}, {}^{x}{\geq}^{y}, {}^{x}{>}^{y}_{c}, {}^{x}{\geq}^{y}_{c} \ | x = m, y \in \{m, M\}\}\rangle$$

They have also provided algorithms to calculate these two unique models and presented a way to combine these models to find a distinguished model of all the types of preferences given together. Their algorithms also capture all the algorithms for handling all the kinds of preferences separately.

It should be pointed out, that the consistency of preference specification, i.e., existence of its preference model, has been assumed by now. This assumption, however, is hard to fulfil in practical applications. In order not to restrict the use of the logic of preference, Boella and Torre [1] have proposed a minimal logic of preference in which *any* preference specification is consistent. They achieve the consistency by means of:

- formalizing a preference φ over ψ as the absence of a ψ world that is preferred over a φ worlds;
- amending the preference model definition by using partial pre-order instead of total pre-order on worlds, which enables to indicate some kind of conflict among worlds (by their incomparability).

Their non-monotonic reasoning is based on distinguished models called *most connected models.*

Definition 7. ***Most connected model*** *[1, Def.4] A model $\mathcal{M} = \langle W, \succeq, \eta\rangle$ is at least as connected as another model $\mathcal{M}' = \langle W, \succeq', \eta\rangle$, written as $\mathcal{M} \sqsubseteq \mathcal{M}'$, if $\succeq' \subseteq \succeq$, i.e.,*

$$\forall w_1, w_2 \in W : w_1 \succeq' w_2 \Rightarrow w_1 \succeq w_2 \ .$$

A model $\mathcal{M}$ is most connected if there is no other model $\mathcal{M}'$ s.t. $\mathcal{M}' \sqsubset \mathcal{M}$, i.e., s.t. $\mathcal{M}' \sqsubseteq \mathcal{M}$ without $\mathcal{M} \sqsubseteq \mathcal{M}'$.

In comparison with Kaci and Torre's language of logic of preferences, their language is by far less expressive, having only one kind of preference.

3 Preferences in database queries

3.1 Basic concepts and key features

To reach the target, we need to accommodate an expressive language with various kinds of preferences in the RDM framework. We propose to base its model-theoretic semantics on those of preference logic languages.

In the following list of basic concepts of our approach, the key features are boldfaced.

– User preferences are expressed in a **preference logic language**.
– Semantics of a set of (possibly conflicting) preferences is related to that of a **disjunctive logic program** (DLP).
– **Non-monotonic reasoning mechanisms** about preferences has to be employed to reason about preferences that are defined in such a way that consistency is ensured under all circumstances.
– A preference operator returning only the best tuples in the sense of user preferences is used to embed preferences into relational query languages.

We identify propositional variables with tuples, i.e., facts over relations. A subset of a relation instance, i.e., a set of facts, creates a world, an element of a set W, and propositions are logically implied by worlds in which they hold true.

3.2 User preferences

Our starting point is the language (Def. 4) introduced by Kaci and Tore [6] who extend propositional language with sixteen kinds of preferences. The aim is to accommodate this expressive language (Def.4) in the RDM framework so that any set of (possibly conflicting) preferences has a well defined semantics.

To define the semantics without the consistency assumption, the definition (Def.2) of the preference model has to be extended. It, however, is not necessary to extend it as much as Boella and Torre [1] have done, who have replaced total pre-order with partial pre-order on worlds in the preference model definition. By contrast, it shows that *partial pre-order*, i.e., a binary relation which is reflexive and transitive, provides a sufficient space of models.

Definition 8. ***(Preference model)*** *A preference model $\mathcal{M}(R) = \langle W, \succeq \rangle$ over a relation instance $I(R)$ is a couple in which $W = \mathcal{P}(I(R))$ is a set of worlds, subsets of relation instance $I(R)$, and $\succeq$ is a partial pre-order over W.*

Observe that as preferences with ceteris paribus provisos can be reduced in accordance with Cor.1 to sets of preferences without such provisos, we have neglected the contextual equivalence.

Definition 9. ***(Models of preferences)*** *Let $\mathcal{M}$ be a preference model, w, w' elements of W, and $w \succ w' := w \succeq w' \wedge \neg(w' \succeq w)$. Then:*

$\mathcal{M} \models \varphi\ {}^{M}{>}^{M}\ \psi$ *iff* $\exists w'$ *s.t.*[2] $\forall w$: *if* $w \models \neg\varphi \wedge \psi$ *and* $\varphi \wedge \neg\psi \not\models_W$ *false, we have* $w' \models \varphi \wedge \neg\psi$ *and* $\neg(w \succeq w')$.
$\mathcal{M} \models \varphi\ {}^{M}{\geq}^{M}\ \psi$ *iff* $\exists w'$ *s.t.* $\forall w$: *if* $w \models \neg\varphi \wedge \psi$ *and* $\varphi \wedge \neg\psi \not\models_W$ *false, we have* $w' \models \varphi \wedge \neg\psi$ *and* $\neg(w \succ w')$.
$\mathcal{M} \models \varphi\ {}^{m}{>}^{M}\ \psi$ *iff* $\forall w \forall w'$: *if* $w \models \neg\varphi \wedge \psi$ *and* $w' \models \varphi \wedge \neg\psi$, *we have* $\neg(w \succeq w')$.
$\mathcal{M} \models \varphi\ {}^{m}{\geq}^{M}\ \psi$ *iff* $\forall w \forall w'$: *if* $w \models \neg\varphi \wedge \psi$ *and* $w' \models \varphi \wedge \neg\psi$, *we have* $\neg(w \succ w')$.
$\mathcal{M} \models \varphi\ {}^{M}{>}^{m}\ \psi$ *iff* $\exists w \exists w'$: *if* $\neg\varphi \wedge \psi \not\models_W$ *false and* $\varphi \wedge \neg\psi \not\models_W$ *false, we have* $w \models \neg\varphi \wedge \psi$, $w' \models \varphi \wedge \neg\psi$, *and* $\neg(w \succeq w')$.
$\mathcal{M} \models \varphi\ {}^{M}{\geq}^{m}\ \psi$ *iff* $\exists w \exists w'$: *if* $\neg\varphi \wedge \psi \not\models_W$ *false and* $\varphi \wedge \neg\psi \not\models_W$ *false, we have* $w \models \neg\varphi \wedge \psi$, $w' \models \varphi \wedge \neg\psi$, *and* $\neg(w \succ w')$.
$\mathcal{M} \models \varphi\ {}^{m}{>}^{m}\ \psi$ *iff* $\exists w \forall w'$: *if* $w \models \neg\varphi \wedge \psi$ *and* $w' \models \varphi \wedge \neg\psi$, *we have* $\neg(w \succeq w')$.
$\mathcal{M} \models \varphi\ {}^{m}{\geq}^{m}\ \psi$ *iff* $\exists w \forall w'$: *if* $w \models \neg\varphi \wedge \psi$ *and* $w' \models \varphi \wedge \neg\psi$, *we have* $\neg(w \succ w')$.

3.3 Preference specification semantics

Definition 10. ***(Preference specification)*** *Let R be a relation schema. Given the set $L_0(R)$ from the definition (Def.4) of the language in which propositional variables are identified with facts over the relation R, $\mathcal{P}_{\rhd}(R)$ is a set of preferences over the relation schema R of the form $\{\varphi_i \rhd \psi_i : i = 1, \ldots, n\}$ for $\varphi_i, \psi_i \in L_0(R)$. A preference specification over the relation schema R is a tuple $\langle \mathcal{P}_{\rhd}(R) | \rhd \in \{ {}^{x}{>}^{y}, {}^{x}{\geq}^{y} | x, y \in \{m, M\}\}\rangle$, and $\mathcal{M}(R)$ is its model iff it models all $\mathcal{P}_{\rhd}(R)$:*

$$\mathcal{M}(R) \models \mathcal{P}_{\rhd}(R) \iff \forall (\varphi_i \rhd \psi_i) \in \mathcal{P}_{\rhd}(R) : \mathcal{M}(R) \models \varphi_i \rhd \psi_i$$

To calculate a preference specification model, we associate it with a DLP in three steps:

First step: Create a partition $(E_1, \ldots, E_n)$ of W so that $w, w' \in E_i$ iff any of the following conditions is fulfilled for every preference $\varphi \rhd \psi$:

1. $w \models \varphi \wedge \neg\psi$ and $w' \models \varphi \wedge \neg\psi$,
2. $w \models \neg\varphi \wedge \psi$ and $w' \models \neg\varphi \wedge \psi$,

[2] $\varphi \wedge \neg\psi \not\models_W$ *false* denotes that there is a model in W for $\varphi \wedge \neg\psi$.

3. $w \models (\varphi \wedge \psi) \vee (\neg\varphi \wedge \neg\psi)$ and
$w' \models (\varphi \wedge \psi) \vee (\neg\varphi \wedge \neg\psi)$.

Second step: Substitute each preference type by a logical formula. In the following list $E_i \models \varphi\wedge\neg\psi$ and $E_j \models \neg\varphi \wedge \psi$ (By abuse of notation, we write $E_i \models \varphi$ iff elements of E_i model φ.)

$\varphi \ {}^{M}\!>^{M} \psi$: $\exists E_i \forall E_j :\not\succeq (E_j, E_i)$ if $\varphi \wedge \neg\psi \not\models_W$ *false*.
$\varphi \ {}^{M}\!\geq^{M} \psi$: $\exists E_i \forall E_j :\not\succ (E_j, E_i)$ if $\varphi \wedge \neg\psi \not\models_W$ *false*.
$\varphi \ {}^{m}\!>^{M} \psi$: $\forall E_j \forall E_i :\not\succeq (E_j, E_i)$.
$\varphi \ {}^{m}\!\geq^{M} \psi$: $\forall E_j \forall E_i :\not\succ (E_j, E_i)$.
$\varphi \ {}^{M}\!>^{m} \psi$: $\exists E_j \exists E_i :\not\succeq (E_j, E_i)$ if $\varphi \wedge \neg\psi \not\models_W$ *false*.
$\varphi \ {}^{M}\!\geq^{m} \psi$: $\exists E_j \exists E_i :\not\succ (E_j, E_i)$ if $\varphi \wedge \neg\psi \not\models_W$ *false*.
$\varphi \ {}^{m}\!>^{m} \psi$: $\exists E_j \forall E_i :\not\succeq (E_j, E_i)$.
$\varphi \ {}^{m}\!\geq^{m} \psi$: $\exists E_j \forall E_i :\not\succ (E_j, E_i)$.

Third step: The above formulae can be expressed in conjunctive normal form. Each of its conjuncts can be represented by an implication: $H_1 \vee \ldots \vee H_n \leftarrow B_1 \wedge \ldots \wedge B_m \wedge \neg B_{m+1} \wedge \ldots \wedge \neg B_{m+k}$, a rule of a DLP.

Furthermore, rules expressing properties of the above predicates and their relations have to be added:

$$\not\succeq (A,B) \vee \succeq (A,B) \leftarrow \not\succeq (B,A),$$
$$\not\succeq (B,A) \vee [\succeq (A,B) \wedge \succeq (B,A)] \leftarrow \not\succ (B,A).$$

$$\succeq (A,C) \leftarrow \succeq (A,B) \wedge \succeq (B,C),$$
$$\parallel (A,B) \leftarrow \not\succeq (A,B) \wedge \not\succeq (B,A),$$
$$\mathit{false} \leftarrow \not\succeq (A,B) \wedge \succeq (A,B).$$

Adding facts: $\succeq (E_1, E_1), \ldots, \succeq (E_n, E_n)$, we finally get the DLP.

3.4 Non-monotonic reasoning

To define the meaning of the program, we employ *optimal model semantics* [7]. First, the formal definition of weight assignment to atoms will be given; then, aggregation strategies will be introduced, and, finally, the optimal models will be defined.

Definition 11. ***(Atomic weight assignment)*** *[7, Def.2] An atomic weight assignment, $\wp$, for a program P, is a map from the Herbrand Base B_P of P to $\mathbb{R}_0^+$, where $\mathbb{R}_0^+$ denotes the set of nonnegative real numbers (including zero).*

Definition 12. ***(Aggregation strategy)*** *[7, Def.3] An aggregation strategy $\mathcal{A}$ is a map from*[3] *$M^{\mathbb{R}_0^+}$ to $\mathbb{R}$.*

Definition 13. ***(Herbrand Objective function)****[7, Def.4] The Herbrand Objective Function, $HOF(\wp, \mathcal{A})$ is a map from $\mathcal{P}(B_P)$ to $\mathbb{R}_0^+$ defined as follows:*

$$HOF(\wp, \mathcal{A})(M) = \mathcal{A}(\{\wp(A) | A \in M\}) \ .$$

[3] Given a set X, M^X denotes the set of all multisets whose elements are in X.

Definition 14. ***(Optimal model)****[7, Def.5] Let P be a logic program, $\wp$ an atomic weight assignment, and $\mathcal{A}$ an aggregation strategy. Suppose that $\mathcal{F}$ is a family of models of P. We say that M is an optimal $\mathcal{F}$-model of P with regard to $(\wp, \mathcal{A})$ if:*

1. *$M \in \mathcal{F}$;*
2. *$\nexists M' : M' \in \mathcal{F} \wedge HOF(\wp, \mathcal{A})(M') < HOF(\wp, \mathcal{A})(M)$.*

We use the notation $Opt(P, \mathcal{F}, \wp, \mathcal{A})$ to denote the set of all optimal $\mathcal{F}$-models of P with regard to $(\wp, \mathcal{A})$.

Applying a variant of the connectivity principle (c.f. Def.7), distinguished models, defining the meaning of the program P, can be selected from stable models $\mathrm{ST}(P)$ of P so that the intensional relation, $\parallel$, of incomparable elements is minimal in the sense of set inclusion. Accordingly, we get the intended optimal model semantics of our program when we extend the notions of aggregation strategy and Herbrand objective function so that the relation of set inclusion can be captured.

It is important to point out that

$$\mathrm{Opt}(P, \mathrm{ST}(P), \wp_0, \mathrm{sum}) \ ,$$

in general, contains more than one optimal model.

3.5 Preference operator

To embed preferences into relational query languages, a *preference operator* $\omega_{\mathcal{P}}$ returning only the best tuples in the sense of user preferences $\mathcal{P}$ is defined.

Ordering the partition of W according to the intensional relation $\succeq$ that is subsumed in an optimal model $M_P \in \mathrm{Opt}(P, \mathrm{ST}(P), \wp_0, \mathrm{sum})$, the most preferred tuples ultimately are located in maximal elements of the partition. To find the maximal elements, the ordered partition is associated with a ground positive datalog program consisting of one rule:

$$M(A) \leftarrow M(B) \wedge \succeq (A, B).$$

and facts: $\succeq (E_i, E_j) \in M_P$.

The least nonempty models of the above positive datalog program yield the interpretations of the predicate M identifying the maximal elements.

3.6 Preferences and relational algebra

The set of algebraic laws that govern the commutativity and distributivity of winnow with respect to relational algebra operations constitutes a formal foundation for rewriting preference queries using the standard strategies like *pushing selection down*.

The following theorem identifies a sufficient condition under which the preference operator and relational algebra selection commute. To improve the readability, $\succeq (x,y) \wedge \neg \succeq (y,x)$ is substituted by $\succ (x,y)$.

Theorem 1 (Commuting selection and the preference operator). *Given a relation schema R, a preference model $\mathcal{M}(R) = \langle W, \succeq \rangle$ where W is a set of all instances over R, a partition $(E_1, \dots, E_n)$ of W ordered by $\succeq$, i.e., $\forall w, w' \in W$ with $w \in E_i, w' \in E_j$ we have $i \leq j \iff w \succeq w'$, and a selection condition φ over R, if the formula*

$$\forall t_1, t_2 : t_1 \in E_i \wedge t_2 \in E_j \wedge \succ (E_j, E_i) \wedge \varphi(t_2) \Rightarrow \varphi(t_2)$$

is valid, then for all instances $I(R)$:

$$\sigma_\varphi(\omega_\mathcal{P}(I(R))) = \omega_\mathcal{P}(\sigma_\varphi(I(R))) \ .$$

To check the validity of the above sufficient condition, we need to assign meaning to the program that consists of rules defining the selection condition $\varphi(x)$, two rules defining $\phi(x)$:

$\phi(x) \leftarrow \varphi(x)$.
$\phi(x) \leftarrow y \in B \wedge x \in A \wedge \succeq (A, B) \wedge \neg \succeq (A, B) \wedge \phi(y)$.

and EDB consisting of the biggest possible instance $I(R)$ of R and facts: $\succeq (E_i, E_j) \in M_P$. The validity of the sufficient condition, then, corresponds to that of the following equality: $\forall t \in I(R) : \varphi(t) = \phi(t)$.

Observe that the above program is *stratifiable*. Thus its stable model semantics can be computed in polynomial time.

4 Conclusions

Pursuing the goal of embedding preference queries in the relational data model, it was shown that **user preferences can be captured in a logical language containing sixteen kinds of preferences**, and the semantics of the language can be defined with respect to the recent advances in logical representation of preferences allowing for **conflicting preferences**. Non-monotonic reasoning about preferences was used to reason about preferences that might be inconsistent as the consistency assumption is hard to fulfill in practical applications.

Embedding preferences into relational query languages was implemented through a preference operator returning the most preferred tuples. This operator has a simple formal semantics defined by means of optimal models of a DLP. A sufficient condition under which the preference operator and relational algebra selection commute was identified, establishing thus a key rule for rewriting the preference queries using the standard algebraic optimization strategies.

Future work directions include developing algorithms for evaluating the preference operator and identification of its algebraic properties, in order to lay the foundation for the optimization of preference queries.

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