A HYBRID KACZMARZ – CG ALGORITHM FOR INCONSISTENT SYSTEMS ARISING IN IMAGE RECONSTRUCTION

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Abstract

Tomographic image reconstruction is the process of reconstructing an object or its cross section from several images of its projections. In the 2D case the object is illuminated by a fan-beam of X-rays, where the signal is attenuated by the object. This leads to a linear system of equations with a sparse matrix, because each observation is influenced only by the pixels on the corresponding beam path. The drawbacks of all Algebraic Reconstruction Techniques (ART) are the computational costs of the iterative formula applied to huge data sets; as an example, in practice the reconstruction of a 256^3 volume and 150 X-ray images of size 1024^2 is a common situation leading us to dimension 16777216 $\times$ 157286400 for the problem matrix. In such cases, the matrix A cannot be any more stored in the computer memory (not even in a compressed form !), thus it has to be re-generated (row by row) during each iteration of an iterative solver. Moreover, because of measurements errors for the X-rays intensities which give us the right-hand side of the problem, the corresponding system of equations becomes also inconsistent. In this paper, we propose a hybrid algorithm for overcoming both storage and inconsistency difficulties mentioned before. It combines in an efficient way an Extended Kaczmarz method from [2] with the general CG algorithm from [1]. We prove convergence of our algorithm for inconsistent linear least squares problems. Numerical experiments are presented on some real medical 2D data sets.

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References

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