bei $[A]$ in beiden Fällen $[x]_1^{m_*} = [0.533333, 0.933334] = -[x]_2^{m_*}$; bei $[\hat{A}]$ gilt jeweils $[\hat{x}]_1^{m_*} = [0.666666, 0.666667] = -[\hat{x}]_2^{m_*}$.

Die Rechnungen wurden auf einem KWS SAM 68K Computer in der Sprache PASCAL-SC [2] durchgeführt.

Literatur

1 ALEFELD, G.; HERZBERGER, J., Introduction to Interval Computations, Academic Press, New York, London 1983.
2 KULISCH, U. W.; MIRANKER, W. L., A New Approach to Scientific Computation, Academic Press, New York, London 1983.
3 MAYER, G., On the asymptotic convergence factor of the total step method in interval computations, Lin. Alg. Appl., (im Druck).
4 MAYER, G., On a theorem of Stein-Rosenberg type in interval analysis, (zur Veröffentlichung eingereicht).
5 ORTEGA, J. M., Numerical Analysis, A Second Course, Academic Press, New York, London 1972.
6 ORTEGA, J. M.; RHEINBOLDT, W. C., Iterative Solution of Nonlinear Equations in Several Varibales. Academic Press, New York, London 1970.
7 VARGA, R. S., Matrix Iterative Analysis, Prentice Hall, Englewood Cliffs, N.J. 1962.

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ROHN, J.

Inverse-Positive Interval Matrices

An $n \times n$ interval matrix $A^I = \{A; \underline{A} \leqq A \leqq \bar{A}\}$ is said to be inverse-positive if $A^{-1} \geqq 0$ for each $A \in A^I$ (the inequality is to be understood componentwise). We give here two new necessary and sufficient conditions for inverse-positivity together with a simple proof of KUTTLER's criterion [1] (assertion (ii) below). A^I is called regular if $\det A \neq 0$ for each $A \in A^I$ and ϱ denotes the spectral radius.

Theorem 1: *The following assertions are equivalent*:

(i) A^I *is inverse-positive*,

(ii) $\bar{A}^{-1} \geqq 0$ *and* $\underline{A}^{-1} \geqq 0$,

(iii) $\bar{A}^{-1} \geqq 0$ *and* $\varrho(\bar{A}^{-1}(\bar{A} - \underline{A})) < 1$,

(iv) $A^{-1} \geqq 0$ *and* A^I *is regular*.

Proof: Denote $D_0 = \bar{A}^{-1}(\bar{A} - \underline{A})$. We shall prove (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i), (i) $\Rightarrow$ (iv) $\Rightarrow$ (iii). (i) $\Rightarrow$ (ii) is obvious. (ii) $\Rightarrow$ (iii): Since $D_0 \geqq 0$ and $(E - D_0)^{-1} = \underline{A}^{-1}\bar{A} = E + \underline{A}^{-1}(\bar{A} - \underline{A}) \geqq 0$, we have $\varrho(D_0) < 1$. (iii) $\Rightarrow$ (i): For each $A \in A^I$ there holds $A = \bar{A}(E - \bar{A}^{-1}(\bar{A} - A))$; since $\varrho(\bar{A}^{-1}(\bar{A} - A)) \leqq \varrho(D_0) < 1$, we have $A^{-1} = \left(\sum_{j=0}^{\infty} (\bar{A}^{-1}(\bar{A} - A))^j\right) \bar{A}^{-1} \geqq 0$. (i) $\Rightarrow$ (iv) is again obvious. (iv) $\Rightarrow$ (iii): Assume that $r = \varrho(D_0) \geqq 1$, then $D_0 x = \bar{A}^{-1}(\bar{A} - \underline{A})\, x = rx$ for some $x \neq 0$, implying $\left(\bar{A} + \frac{1}{r}(\underline{A} - \bar{A})\right) x = 0$, hence $\bar{A} + \frac{1}{r}(\underline{A} - \bar{A}) \in A^I$ is singular, a contradiction. This completes the proof.

Thus we have also proved the following result.

Theorem 2: *Let A^I be inverse-positive. Then for each $A \in A^I$ we have* $A^{-1} = \left(\sum_{j=0}^{\infty} (\bar{A}^{-1}(\bar{A} - A))^j\right) \bar{A}^{-1}$.

Let us recall that, given an interval n-vector $b^I = [\underline{b}, \bar{b}]$, the exact interval solution $x^I = [\underline{x}, \bar{x}]$ of the system of interval linear equations $A^I x = b^I$ is defined by $\underline{x}_i = \min_X x_i$, $\bar{x}_i = \max_X x_i$ $(i = 1, \dots, n)$, where $X = \{x; Ax = b, A \in A^I, b \in b^I\}$. For inverse-positive matrices A^I satisfying $\bar{A}^{-1} > 0$, the exact interval solution can be computed by an iterative method. For the purpose of its formulation, denote $A_c = \frac{1}{2}(\underline{A} + \bar{A})$, $\Delta = \frac{1}{2}(\bar{A} - \underline{A}) \geqq 0$, and for any $x = (x_i) \in R^n$, let $|x| = (|x_i|)$.

Theorem 3: *Let $\bar{A}^{-1} > 0$ and $\underline{A}^{-1} > 0$. Then, we have $\underline{x} = \lim_{m \to \infty} \underline{x}_m$, $\bar{x} = \lim_{m \to \infty} \bar{x}_m$, where the sequences $\{\underline{x}_m\}_0^\infty$, $\{\bar{x}_m\}_0^\infty$ are given by*

$$\underline{x}_0 = A_c^{-1}\underline{b}\,, \qquad \underline{x}_{m+1} = -A_c^{-1}\Delta|\underline{x}_m| + A_c^{-1}\underline{b} \quad (m = 0, 1, 2, \dots)$$

and

$$\bar{x}_0 = A_c^{-1}\bar{b}\,, \qquad \bar{x}_{m+1} = A_c^{-1}\Delta|\bar{x}_m| + A_c^{-1}\bar{b} \quad (m = 0, 1, 2, \dots)\,.$$

For the proof of this theorem, see [3]. The convergence of both the sequences is guarranteed by the next theorem, which can be proved in the same way as implication (iv) ⇒ (iii) of theorem 1.

Theorem 4: *Let A^I be inverse-positive. Then $\varrho(A_c^{-1}\Delta) < 1$.*

Under an additional assumption, the exact interval solution x^I can be expressed explicitly. This is a generalization of the result by BARTH and NUDING [2] (their condition $\underline{b} \geq 0$ implies $\bar{A}^{-1}\underline{b} \geq 0$).

Theorem 5: *Let A^I be inverse-positive and let $\bar{A}^{-1}\underline{b} \geq 0$. Then we have $\underline{x} = \bar{A}^{-1}\underline{b}$, $\bar{x} = \underline{A}^{-1}\bar{b}$.*

Proof: For each $A \in A^I$, $b \in b^I$, theorem 2 implies $A^{-1}b = \left(\sum_{j=0}^{\infty} (\bar{A}^{-1}(\bar{A} - A))^j\right) \bar{A}^{-1}b \geq \bar{A}^{-1}\underline{b}$, hence $\underline{x} = \bar{A}^{-1}\underline{b}$. Secondly, if $Ax = b$ for some $A \in A^I$, $b \in b^I$, then $x \geq \underline{x} \geq 0$, hence $\underline{A}x \leq Ax = b \leq \bar{b}$ and premultiplying by $\underline{A}^{-1}$ yields $x \leq \underline{A}^{-1}\bar{b}$; hence $\bar{x} = \underline{A}^{-1}\bar{b}$.

References

1 KUTTLER, J., A Fourth-Order Finite-Difference Approximation for the Fixed Membrane Eigenproblem, Math. of Computation 25, 237–256 (1971).
2 BARTH, W.; NUDING, E., Optimale Lösung von Intervallgleichungssystemen, Computing 12, 117–125 (1974).
3 ROHN, J., Solving Interval Linear Systems, preprint, Freiburger Intervall-Berichte 84/7, 1–14.
4 ROHN, J., Interval Linear Systems, preprint, Freiburger Intervall-Berichte 84/7, 33–58.

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SCHMITT, B. A.

A Defect Correction Method on $(-\infty, \infty)$

The boundary value problem

$$-u'' + A(t)\,u = g(t)\,, \qquad t \in (-\infty, \infty)\,, \qquad u \to 0\,, \qquad t \to \pm\infty \tag{1}$$

will be considered. We assume $A(t) \to A_\infty$, $t \to \pm\infty$, $\operatorname{Re} A_\infty := \frac{1}{2}(A + A^\mathsf{T}) \geq a^2 > 0$, A, g smooth. Our aim is the computation of high accuracy solutions.

In the treatment of problem (1) two kinds of discretizations are needed. The first has to deal with the differential equation. Here, good approximations are possible e.g. with high order difference expansions of the second derivative d^2/dt^2. The simplest is $(D_{2,h}y)_i := h^{-2}[y_{i-1} - 2y_i + y_{i+1}]$ and leads to the system

$$-D_{2,h}y_i + A(t_i)\,y_i = g_i := g(t_i)\,, \qquad t_i := ih\,, \qquad i \in Z\,. \tag{2}$$

This ordinary difference method has order h^2.

The second approximation is needed for the infinite interval $(-\infty, \infty)$. This is usually truncated by introducing additional boundary conditions at finite endpoints (here we consider only the right hand end, $t = t_r$), for instance the simple condition $u(t_r) = 0$, with an error $O(e^{-\alpha t_r} u(t_r))$, $t_r \to \infty$, or the asymptotically optimal condition $u'(t_r) + (A_\infty)^{1/2} u(t_r) = 0$ of DE HOOG and WEISS [4] and LENTINI and KELLER [5] which introduces an error $O(e^{-\alpha t_r} g(t_r))$, $t_r \to \infty$, with respect to any given point t.

For differential equations the construction of high order approximations is well understood. But the cited boundary conditions give only "first order" approximations with respect to the behaviour of $g(t_r)$, $t_r \to \infty$.

First, we consider the construction of discrete boundary conditions in the constant coefficient case.

Lemma 1: *Let the coefficient matrix A be constant,* $\operatorname{Re} A \geq \alpha^2 > 0$. *Then*

a) *There exists a unique matrix M, $\|M^{-1}\| < 1$ with $M + M^{-1} = 2I + h^2A$.*

b) *At any index r the solution y of* (2) *satisfies the equation*

$$-y_{r-1} + My_r = h^2 \sum_{i=0}^{\infty} M^{-j} g_{r+j} =: G_r\,. \tag{3}$$

Remark: By using equation (3), and a similar one at an index $l < r$, as discrete boundary conditions for the infinite discrete problem (2) its "exact" solution can be computed by solving a finite system with block-tridiagonal matrix.

Surely G_r in (3) is not known, but one can compute it with high accuracy by a series transformation, e.g. the ε-algorithm, which, under suitable assumptions, has for a fixed number $2k$ of coefficients g_j in (3) an error $O(e^{-\alpha t} g(t_r)^k)$, $t_r \to \infty$. This can be derived from a result of GRAGG [3].

Thus (3) gives an high order approximation for the interval $(-\infty, \infty)$, too. ■