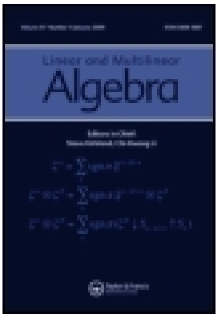

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Letter to the editor

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Letter to the editor

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It is shown that the recent characterization of strong feasibility of interval linear equations by Karademir and Prokopyev was in fact published by this author 13 years earlier. The original result was difficult to discover, having been embedded, together with its proof, into the proof of another theorem.

Keywords: interval linear equations; strong feasibility; Farkas-type theorem

AMS Subject Classifications: 15A06; 65G40

Recently, Karademir and Prokopyev [2] published a Farkas-type theorem for strong feasibility of interval linear equations. They stated without proof that a system of interval linear equations $\mathbf{A}x=\mathbf{b}$ with $\mathbf{A}=[A_c-\Delta, A_c+\Delta]$ and $\mathbf{b}=[b_c-\delta, b_c+\delta]$ is strongly feasible if and only if the system

$$A_c^T y + \Delta^T |y| \geq 0, \tag{1}$$

$$b_c^T y - \delta^T |y| < 0 \tag{2}$$

has no solution (where the absolute value of a vector is taken entrywise). By definition [1], a system $\mathbf{A}x=\mathbf{b}$ is called strongly feasible if $Ax=b$ has a nonnegative solution for each $A \in \mathbf{A}$, $b \in \mathbf{b}$. In [2], the authors made the comment 'The proof is similar to the proof of Proposition 1 [2] and is omitted here'.

Karademir and Prokopyev [2] were not aware, however, of the fact that both this result and its proof were published in [3] as part (1) of the proof of Theorem 1, included as an auxiliary tool for proving a statement of NP-hardness of linear programming with inexact data. So, the result was not stated in [3] as an independent theorem, having been embedded, together with its proof, into the proof of another theorem. No wonder, therefore, that the result went unnoticed.

In what follows, we give a verbatim copy of the result and its proof as it was published in [3] as part (1) of the proof of Theorem 1 there. The quoted part is included into quotation marks.

'(1) First we prove that each system

$$Ax = b, \quad x \geq 0 \tag{3}$$

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with data satisfying

$$A \in \mathbf{A}, \quad b \in \mathbf{b} \tag{4}$$

has a solution if and only if

$$(\forall y)\left(A_c^T y + \Delta^T |y| \geq 0 \Rightarrow b_c^T y - \delta^T |y| \geq 0\right) \tag{5}$$

holds. "Only if": Let each system (3) with data (4) have a solution, and let $A_c^T y + \Delta^T |y| \geq 0$ for some $y \in \mathbb{R}^m$. Define a diagonal matrix T by $T_{ii} = 1$ if $y_i \geq 0$, $T_{ii} = -1$ if $y_i < 0$, and $T_{ij} = 0$ if $i \neq j$ $(i, j = 1, \ldots, m)$, then $|y| = Ty$. Consider now the system

$$(A_c + T\Delta)x = b_c - T\delta, \quad x \geq 0. \tag{6}$$

Since $A_c + T\Delta \in \mathbf{A}$ and $b_c - T\delta \in \mathbf{b}$, the system (6) has a solution according to the assumption, and $(A_c + T\Delta)^T y = A_c^T y + \Delta^T |y| \geq 0$, hence Farkas lemma applied to (6) gives that $b_c^T y - \delta^T |y| = (b_c - T\delta)^T y \geq 0$, which proves (5). "If": Assuming that (5) holds, consider a system (3) with data satisfying (4). Let $A^T y \geq 0$ for some y; then $A_c^T y + \Delta^T |y| \geq (A_c + A - A_c)^T y = A^T y \geq 0$, hence (5) gives that $b^T y = (b_c + b - b_c)^T y \geq b_c^T y - \delta^T |y| \geq 0$. Thus, we have proved that for each y, $A^T y \geq 0$ implies $b^T y \geq 0$, and Farkas lemma proves the existence of a solution to (3)'.

Obviously, (5) is equivalent to the nonexistence of a solution to (1), (2).

The fact that Karademir and Prokopyev [2] were unaware of the result in [3] is not surprising, for the result was so well-hidden that it was hard to discover in [3]. Moreover, it was not included into the survey chapter in [1] in order not to disturb the unified approach there.

This author is embarrassed to admit that while acting as a reviewer of this article [2], he did not recognize his own result there; the fact came to him only later.

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