Reliable Computing (2006) **12:** 245–246
DOI: 10.1007/s11155-006-7222-7

Letter to the Editor

JIŘÍ ROHN
Institute of Computer Science, Czech Academy of Sciences, Pod vodárenskou věží 2, 182 07 Prague, Czech Republic, e-mail: rohn@cs.cas.cz

(Received: 25 October 2005)

E. Hansen has recently published in [2] a criterion for regularity of interval matrices. As is well known, an $n \times n$ interval matrix $\mathbf{A} = [\underline{A}, \overline{A}]$ is called regular if each $A \in \mathbf{A}$ is nonsingular. Hansen first defines vertex matrices in $\mathbf{A}$ as matrices satisfying

$$A_{ij} \in \{\underline{A}_{ij}, \overline{A}_{ij}\} \qquad (i, j = 1, \dots, n). \tag{1}$$

His result is then formulated as follows: *An interval matrix* $\mathbf{A}$ *is regular if and only if the determinants of all the vertex matrices in* $\mathbf{A}$ *are nonzero and of the same sign.* If $\underline{A} < \overline{A}$, then according to (1) for each i, j we have two options for choosing A_{ij}, hence there are 2^{n^2} mutually different vertex matrices in this case. So, in the worst case Hansen's criterion requires evaluation of determinants of 2^{n^2} matrices.

This result, however, brings nothing new and is in fact much worse than a criterion published more than 20 years ago by my former undergraduate student M. Baumann [1]; the result has been reproved and republished in [6], Theorem 5.1, assertion (C1), and in the monograph [4], Theorem 21.4, proof on pp. 266–267 (Hansen's paper [2] contains no references whatsoever!). Baumann's criterion employs matrices A_{yz} defined for each ± 1-vectors $y, z \in \mathbb{R}^n$ in the following way:

$$(A_{yz})_{ij} = \begin{cases} \overline{A}_{ij} & \text{if } y_i z_j = -1, \\ \underline{A}_{ij} & \text{if } y_i z_j = 1, \end{cases} \qquad (i, j = 1, \dots, n), \tag{2}$$

and sounds verbally alike: $\mathbf{A}$ *is regular if and only if the determinants of all the matrices* A_{yz} *(where* y, z *are* ± 1*-vectors in* $\mathbb{R}^n$*) are nonzero and of the same sign.* Since there are 2^{2n} pairs of $\pm$ 1-vectors y, z and since $A_{-y,-z} = A_{yz}$ for each such y, z, only 2^{2n-1} determinants are to be evaluated according to Baumann's criterion, compared to 2^{n^2} of them by that of Hansen. Moreover, each matrix A_{yz} satisfies (1) and is thereby a vertex matrix, so that Baumann's criterion employs a *subset* of Hansen's vertex matrices. The reduction from 2^{n^2} to 2^{2n-1}, although both the values are exponential in n, is essential: e.g. for $n = 10$ it brings 2^{100} down to 2^{19}.

It is amusing that in the same issue of the *Reliable Computing* journal V. Kreinovich [3] showed that Baumann's criterion is optimal in the following sense: for each fixed pair of $\pm$ 1-vectors $\tilde{y}, \tilde{z}$ one can find an interval matrix $\mathbf{A}$

such that the determinants of all the matrices A_{yz} for $(y, z) \neq (\tilde{y}, \tilde{z})$, $(y, z) \neq (-\tilde{y}, -\tilde{z})$ are nonzero and of the same sign, but $\mathbf{A}$ is not regular. Hence, Baumann's criterion cannot be generally improved. The fact that it is exponential is explained by NP-hardness of checking regularity of interval matrices proved in [5].

References

1. Baumann, M.: A Regularity Criterion for Interval Matrices, in: Garloff, J. et al. (eds), *Collection of Scientific Papers Honouring Prof. Dr. K. Nickel on Occasion of His 60th Birthday*, Part I, Freiburg University, Freiburg, 1984, pp. 45–50.
2. Hansen, E.: A Theorem on Regularity of Interval Matrices, *Reliable Computing* **11** (6) (2005), pp. 495–497.
3. Kreinovich, V.: Optimal Finite Characterization of Linear Problems with Inexact Data, *Reliable Computing* **11** (6) (2005), pp. 479–489.
4. Kreinovich, V., Lakeyev, A., Rohn, J., and Kahl, P.: *Computational Complexity and Feasibility of Data Processing and Interval Computations*, Kluwer Academic Publishers, Dordrecht, 1998.
5. Poljak, S. and Rohn, J.: Checking Robust Nonsingularity Is NP-Hard, *Mathematics of Control, Signals, and Systems* **6** (1993), pp. 1–9.
6. Rohn, J.: Systems of Linear Interval Equations, *Linear Algebra and Its Applications* **126** (1989), pp. 39–78.