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Introduction

The usual "implementation" of real numbers as floating point numbers on existing computers has the well-known disadvantage that most of the real numbers are not exactly representable in floating point. Also the four basic arithmetic operations can usually not be performed exactly.

For numerical algorithms there are frequently error bounds for the computed approximation available. Traditionally a bound for the infinity norm is estimated using theoretical concepts like the condition number of a matrix for example. Therefore the error bounds are not really available in practice since their computation requires more or less the exact solution of the original problem.

During the last years research in different areas has been intensified in order to overcome these problems. As a result applications to different concrete problems were obtained.

The LEDA-library (K. Mehlhorn et al.) offers a collection of data types for combinatorical problems. In a series of applications, where floating point arithmetic fails, reliable results are delivered. Interesting examples can be found in classical geometric problems.

At the Imperial College in London was introduced a simple principle for "exact arithmetic with real numbers" (A. Edalat et al.), which uses certain nonlinear transformations. Among others a library for the effective computation of the elementary functions already has been implemented.

Using symbolic-algebraic methods the solution of a given problem can be computed exactly. These methods are applied successfully in many fields. However, for large problems the computing time may become prohibitive.

Another possibility is offered by so-called verification methods. These methods give correct results using only floating point arithmetic. Error bounds are computed by a sophisticated combination of error estimators. This idea allows to attack even larger problems without loosing too much time in comparison to traditional methods (without verification).

During the last few years it was already started to combine symbolic-algebraic methods and verification methods to so-called hybrid methods.

Scientists in different fields are working today on the outlined subjects. It was the purpose of a Dagstuhl seminar (with the same title as this book) at the Forschungszentrum für Informatik, Schloß Dagstuhl, Germany, to bring together colleagues from Computer Science, Computer Algebra, Numerical Mathematics, Matrix- and NP-theory, Control Theory and similar fields for exchanging the latest results of research and ideas.

This book contains (in alphabetical order) a collection of worked-out talks presented during this seminar. All contributions have been refereed. We are thankful to the authors for submitting their papers and to the referees for assisting us.

We would like to express our warmest thanks to Professor Dr. Reinhard Wilhelm for giving us the opportunity to run this seminar in Dagstuhl and to the whole crew of Schloß Dagstuhl for presenting a very nice atmosphere which let all participants feel like at home.

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