Statistical modelling in climate science

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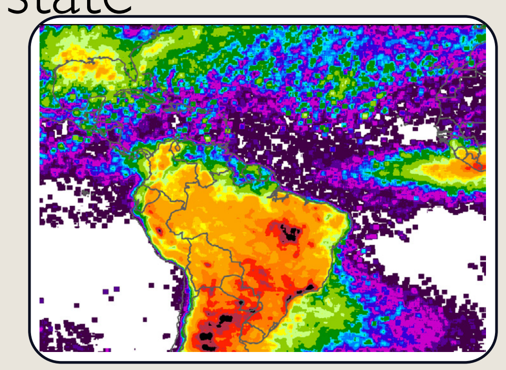
Introduction modelling in climate science

dynamical model

initial state



eqs.

$$-\eta\nabla^2\mathbf{u} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{F}$$

$$\nabla\cdot\mathbf{u} = 0$$

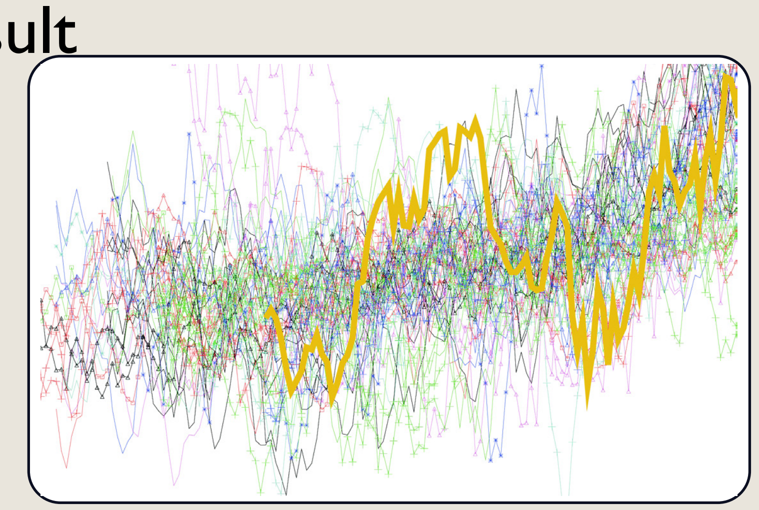
$$\frac{\partial}{\partial t}\left[\rho\left(e+\frac{\mathbf{u}^2}{2}\right)\right] + \nabla\cdot\left[\rho\left(e+\frac{\mathbf{u}^2}{2}\right)\mathbf{u}\right] =$$

$$\rho\dot{q} - \frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} + \rho\mathbf{F}\cdot\mathbf{u}$$

$$\dots$$

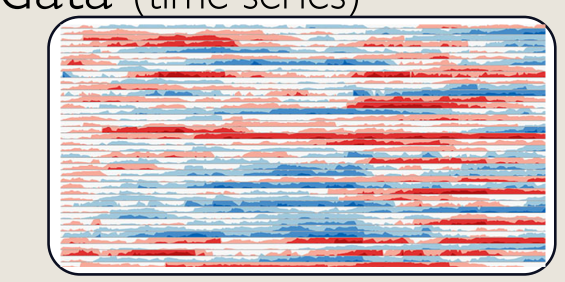
$$\Downarrow\Downarrow \int \dots \mathrm{d}t$$

result



statistical model

past data (time series)



$\Downarrow$ train \ estimate

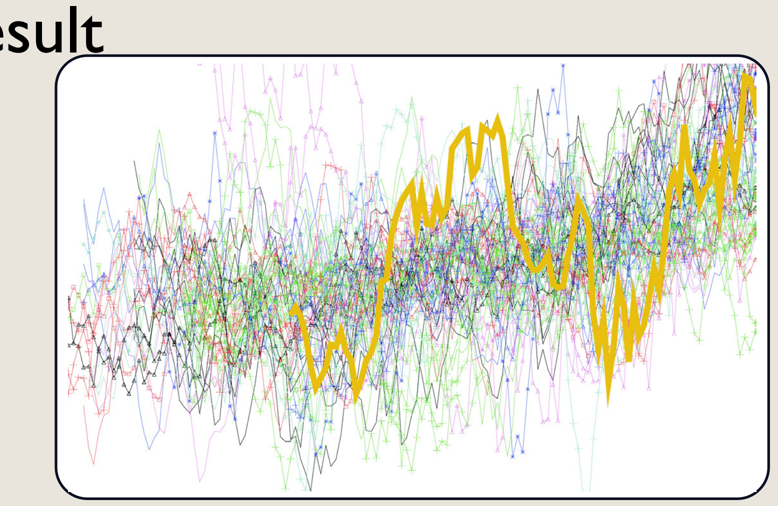
stat. model

$$\mathrm{d}x_i = \left(\mathbf{x}^T\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\mathbf{x} + c_i\right)\mathrm{d}t + \mathrm{d}r_i$$

$$\mathrm{d}r^{(L)} = \mathbf{b}_i^{(L)}\left[\mathbf{x}, \mathbf{r}^{(0)}, \dots, \mathbf{r}^{(L)}\right]\mathrm{d}t + \mathrm{d}r_i^{(L+1)}$$

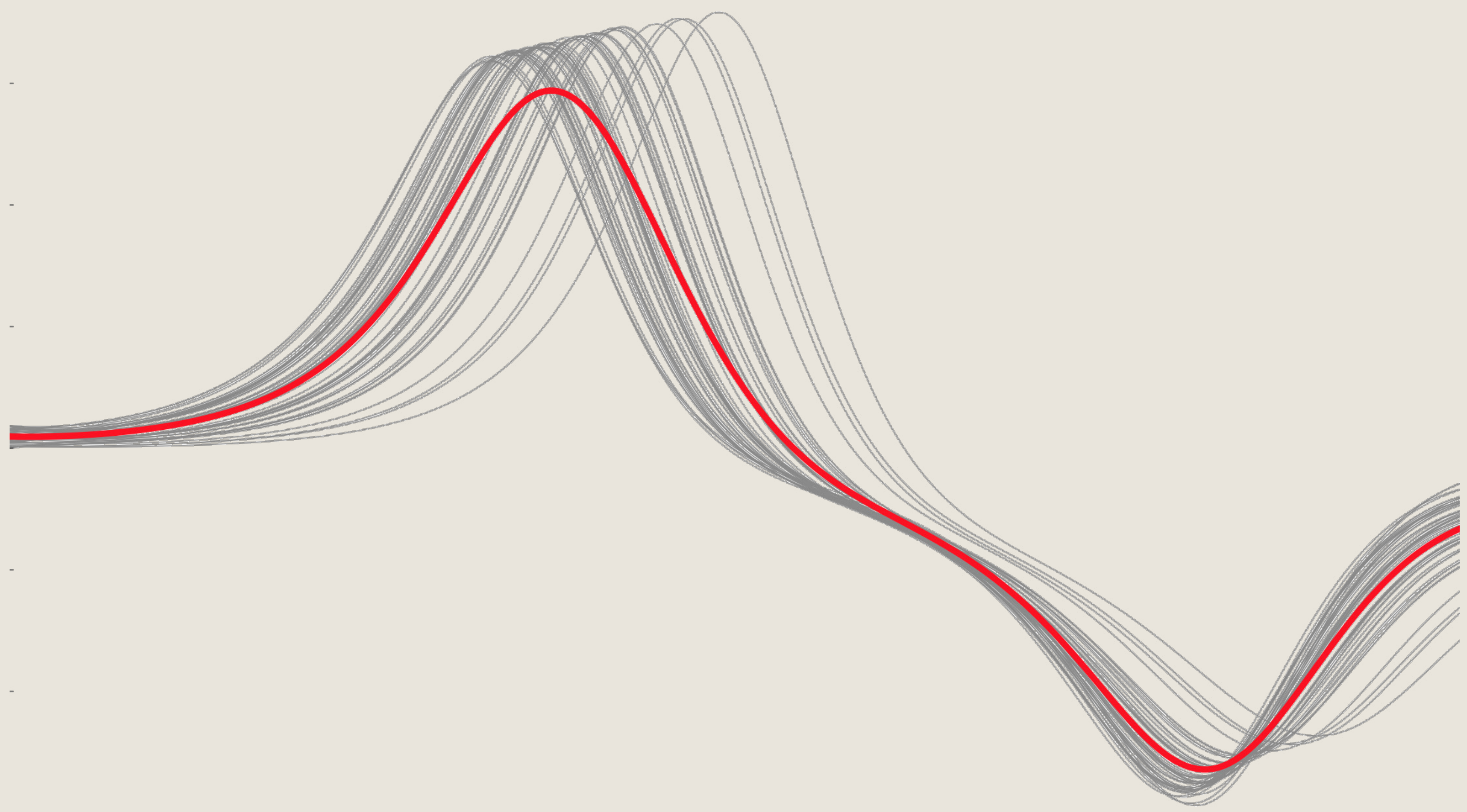
$$\Downarrow\Downarrow \int \dots \mathrm{d}t$$

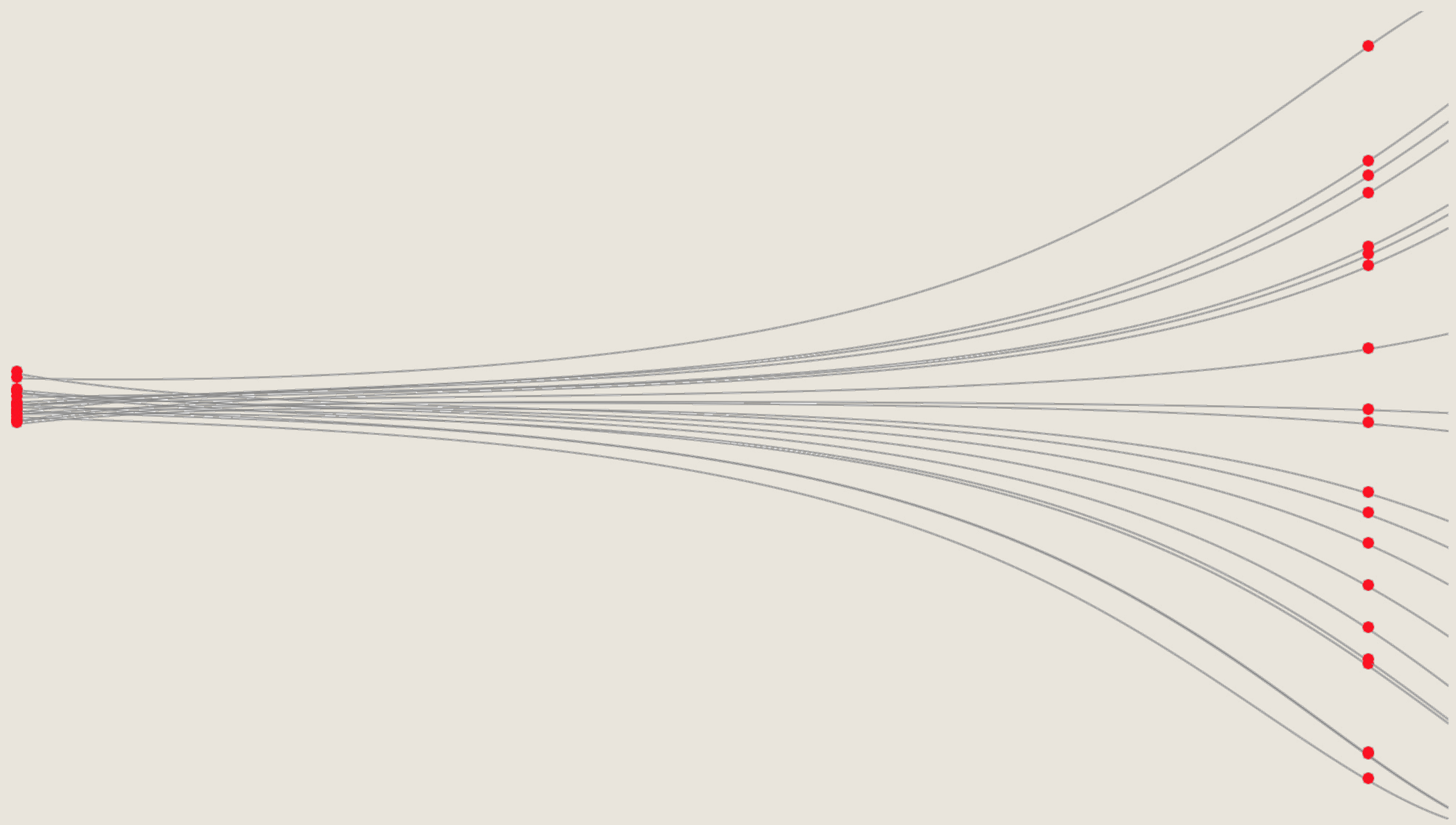
result



Introduction dynamical models

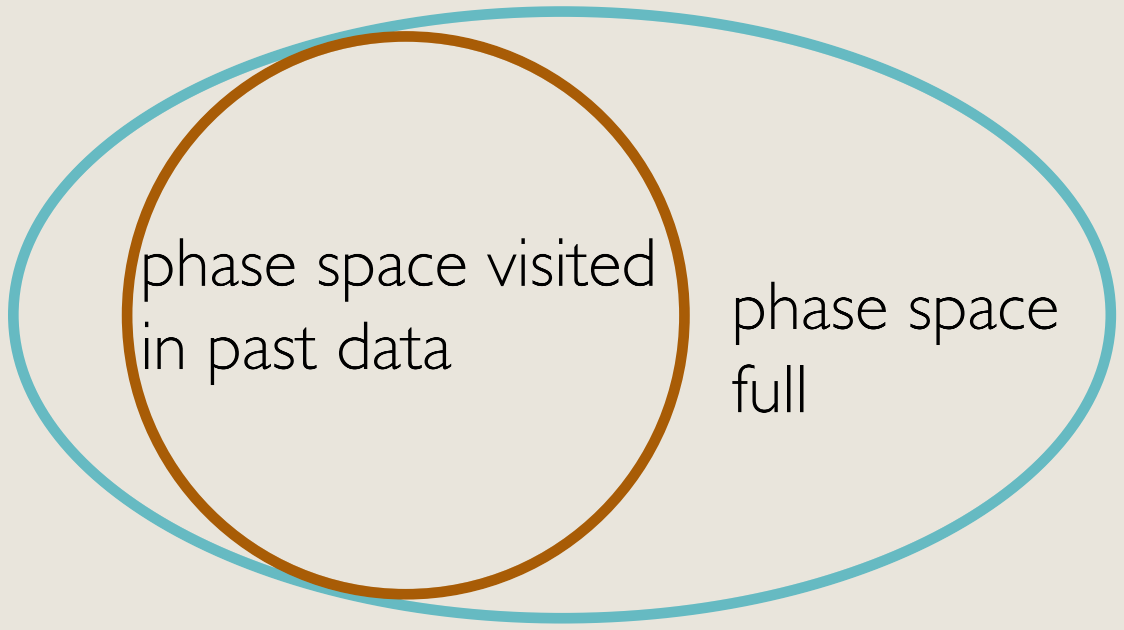
- discretized partial differential equations + current state of the climate (initial cond.)
- general circulation models (GCMs)
- used in numerical weather prediction, reanalysis data-sets and future climate intercomparisons
- uncertainties: initial errors and model errors





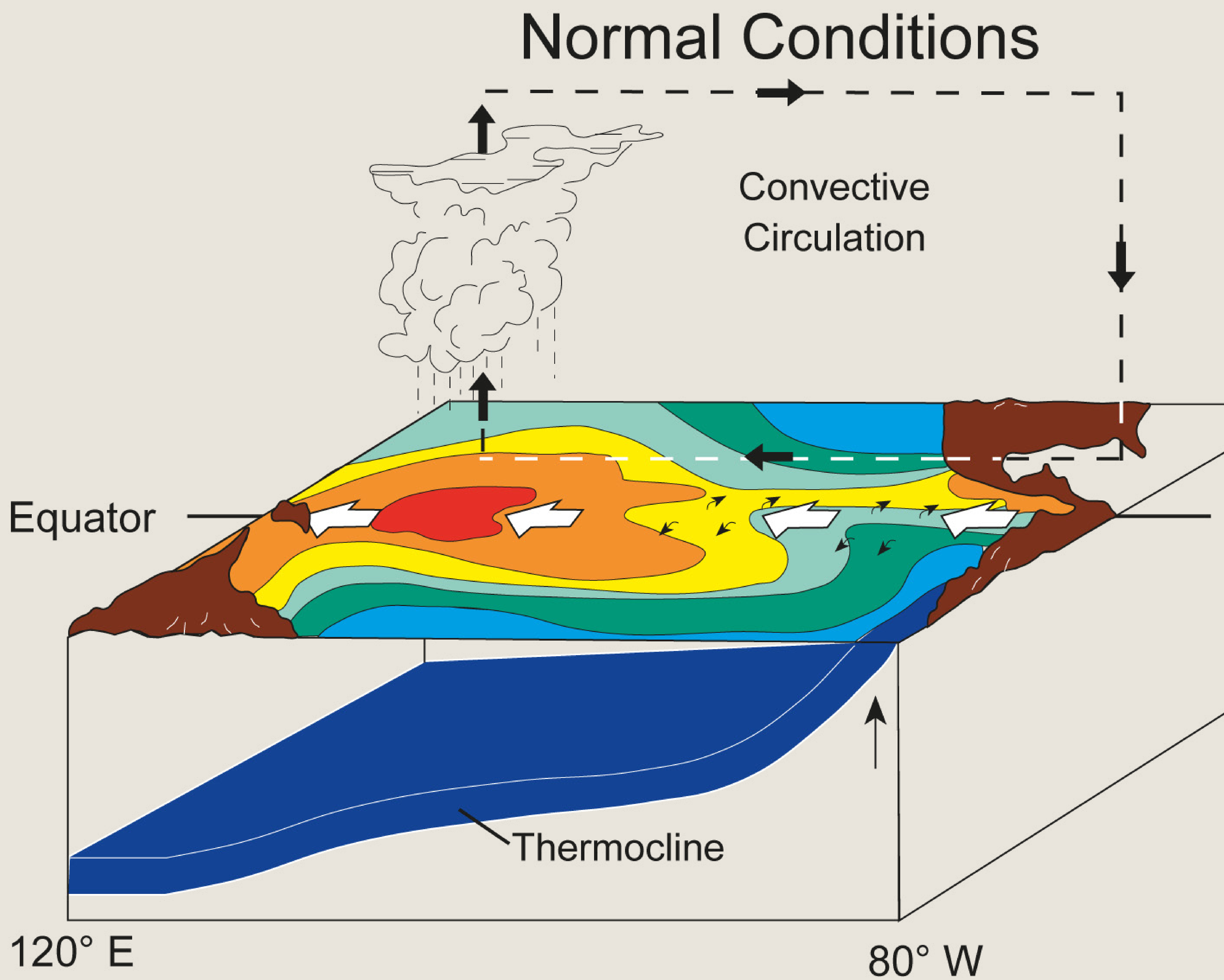
Introduction statistical models

- motivation: forecasting, scaling down the complexity
- NOT based on physical mechanisms underlying the dynamics, but derived from past weather patterns
- inverse stochastic models
- model is designed, its parameters estimated / trained using past weather data and stochastically integrated
- uncertainties: which variables and non-stationarity
  - temperature?
  - atm. pressure?
  - sea-surface temperature?
  - cloudiness?
  - latent heat flux?



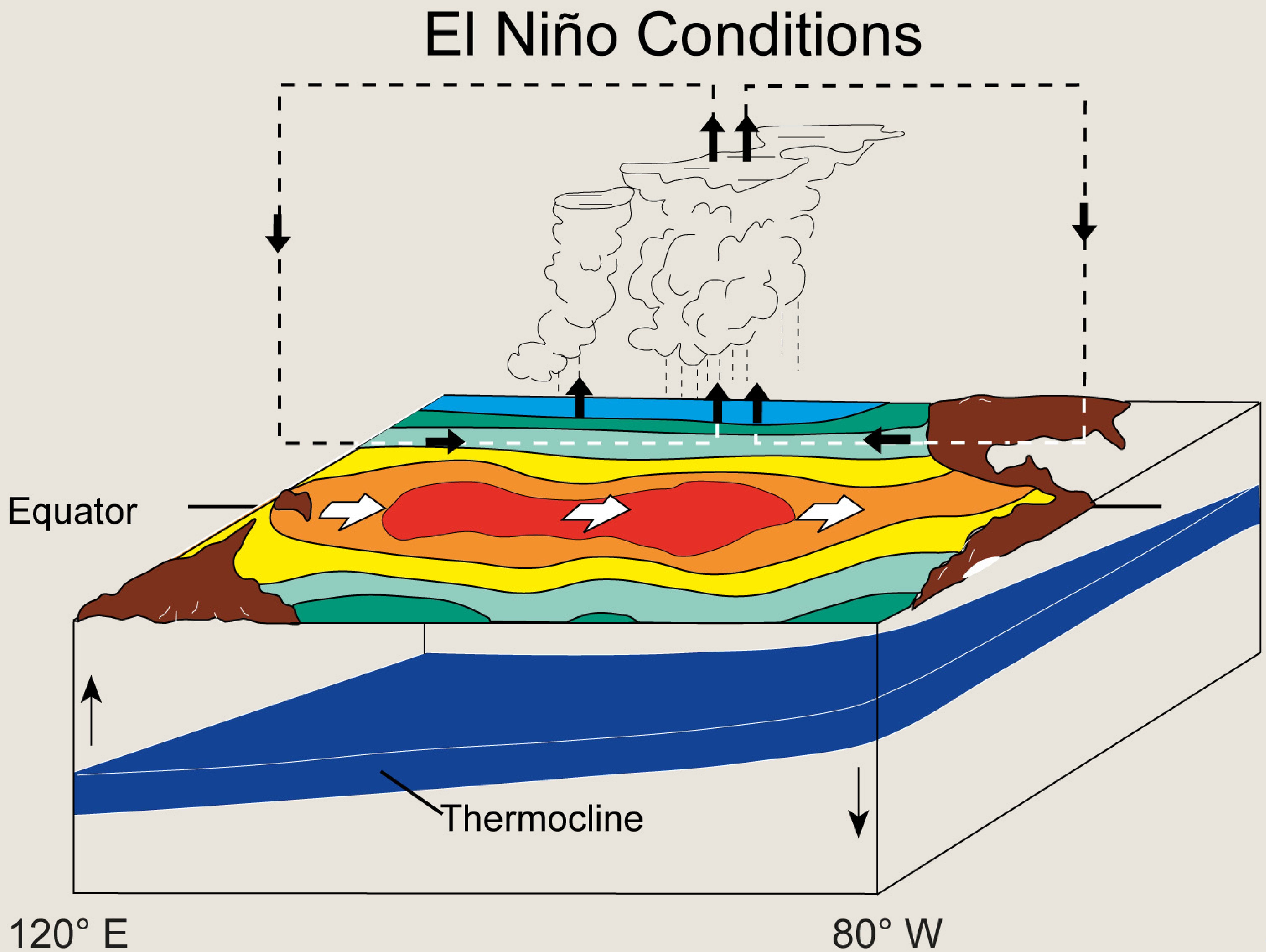
ENSO neutral

- strong interannual signal with great economic and so-cietal impact



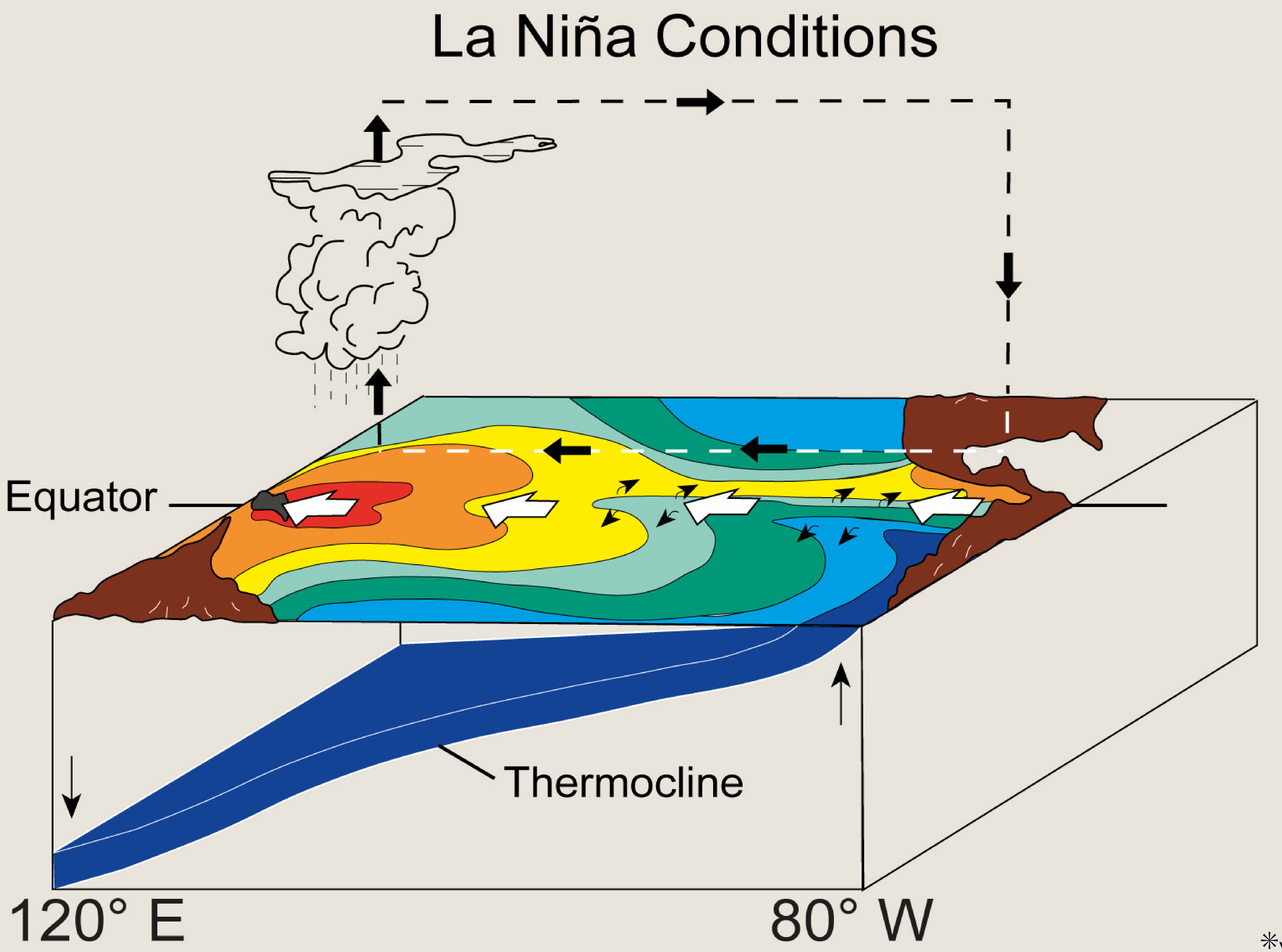
*wikipedia.org

ENSO positive



*wikipedia.org

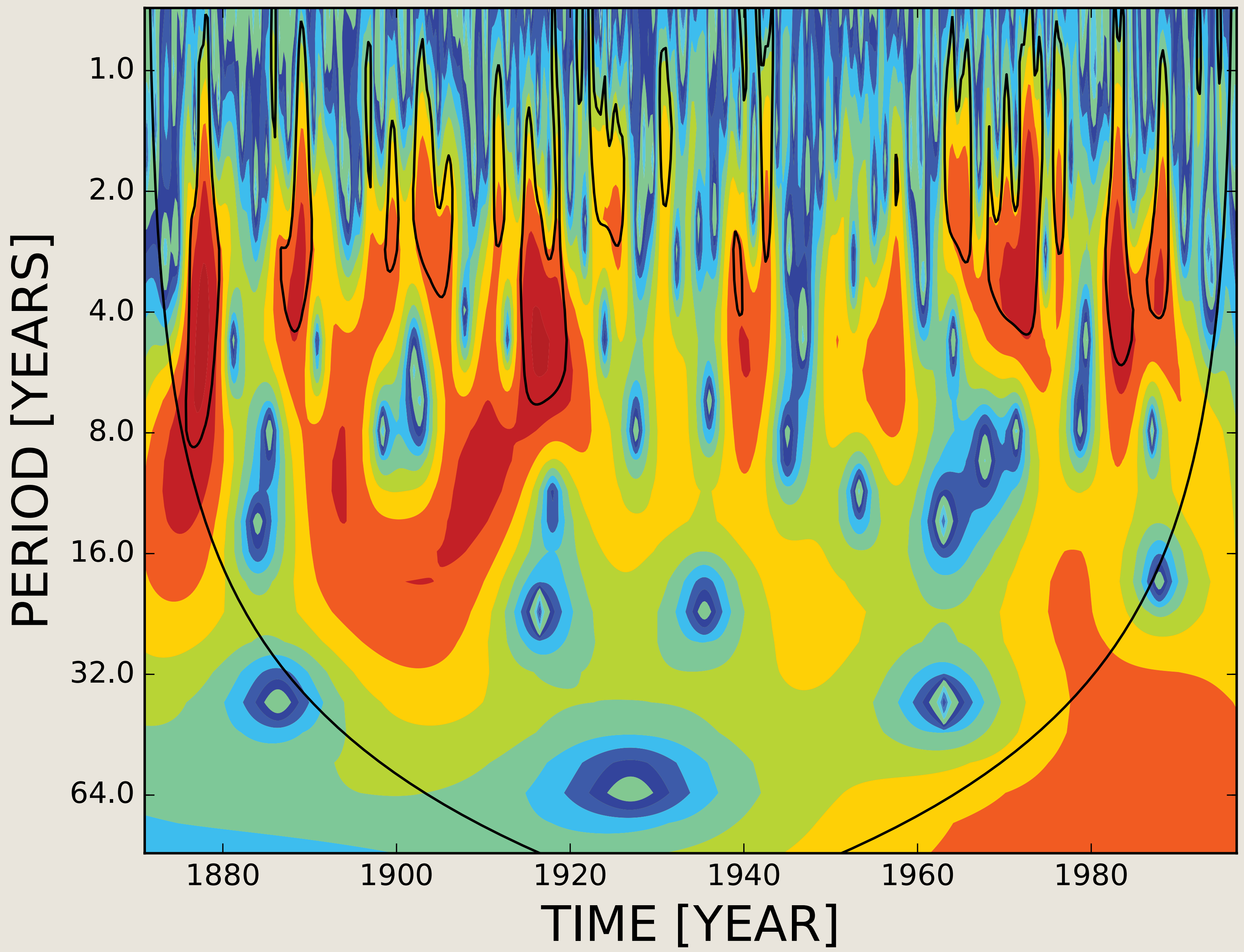
ENSO negative



*wikipedia.org

ENSO overview

- naturally oscilates between phases without a distinct period



- reasons why are still largely unknown
- positive phase characterised by a larger magnitude than negative phase -- nonlinear interactions

Statistical model inverse nonlinear model

- evolution of anomalies as
  $\dot{\mathbf{x}} = \mathbf{L}\mathbf{x} + \mathbf{N}(\mathbf{x})$
- linear inverse models by assuming linear form
  $\mathbf{N}(\mathbf{x})\mathrm{d}x \approx \mathbf{T}\mathbf{x}\mathrm{d}t + \mathrm{d}\mathbf{r}^{(0)}$
- describes linear feedback of hidden processes
- assume polynomial form
  $N_i(\mathbf{x})\mathrm{d}x \approx (\mathbf{x}^T\mathbf{A}_i\mathbf{x} + \mathbf{t}_i\mathbf{x} + c_i^{(0)})\mathrm{d}t + \mathrm{d}r_i^{(0)}$
  $\mathbf{b}_i^{(0)} = \mathbf{l}_i + \mathbf{t}_i,\ \mathbf{B}^{(0)} = \mathbf{L} + \mathbf{T}$
- so that the main level of our model is
  $\mathrm{d}x_i = \left(\mathbf{x}^T\mathbf{A}_i\mathbf{x} + \mathbf{b}_i^{(0)} + c_i^{(0)}\right)\mathrm{d}t + \mathrm{d}r_i^{(0)}$

Statistical model multilevel models

- stochastic forcing still involves serial correlations and might also depend on modelled process
- additional levels included to express the known time increments as linear function of extended state vector

$$\mathrm{d}r_i^{(0)} = \mathbf{b}_i^{(1)}\left[\mathbf{x}, \mathbf{r}^{(0)}\right]\mathrm{d}t + r_i^{(1)}\mathrm{d}t$$
$$\dots$$
$$\mathrm{d}r_i^{(L)} = \mathbf{b}_i^{(L+1)}\left[\mathbf{x}, \mathbf{r}^{(0)}, \dots, \mathbf{r}^{(L)}\right]\mathrm{d}t + r_i^{(L+1)}\mathrm{d}t$$

Statistical model ENSO model

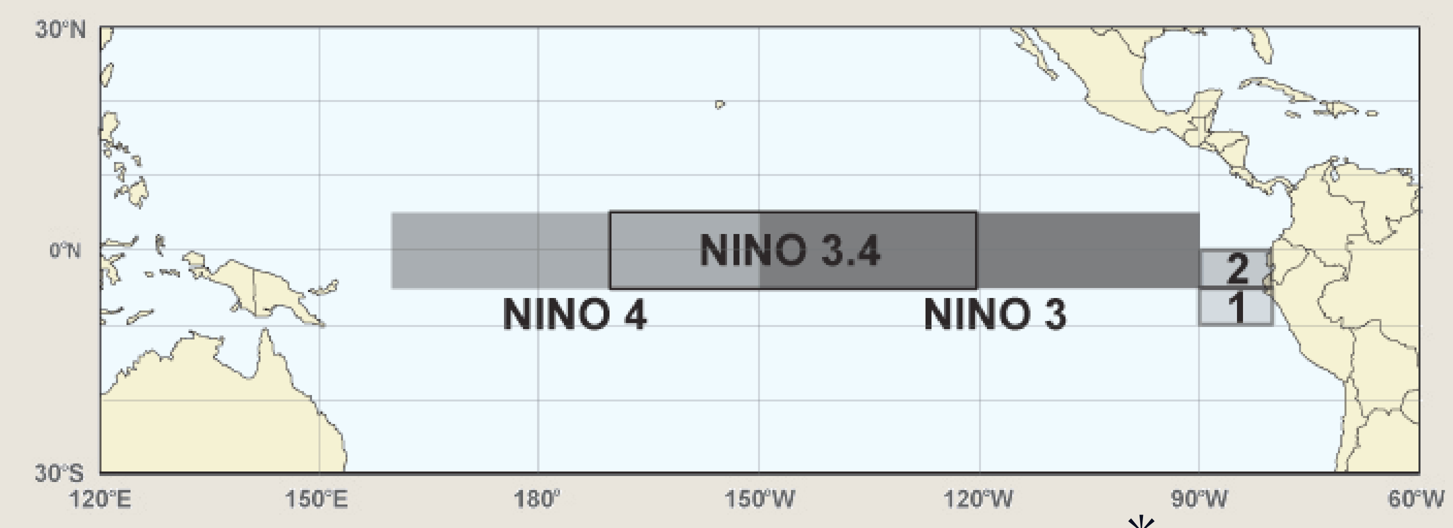
- it is known that extreme ENSO events tend to occur in boreal winter, we include seasonality as

$$\mathbf{B}^{(0)} = \mathbf{B}_0 + \mathbf{B}_s \sin(2\pi t/T) + \mathbf{B}_c \cos(2\pi t/T)$$
$$\mathbf{c}^{(0)} = \mathbf{c}_0 + \mathbf{c}_s \sin(2\pi t/T) + \mathbf{c}_c \cos(2\pi t/T)$$

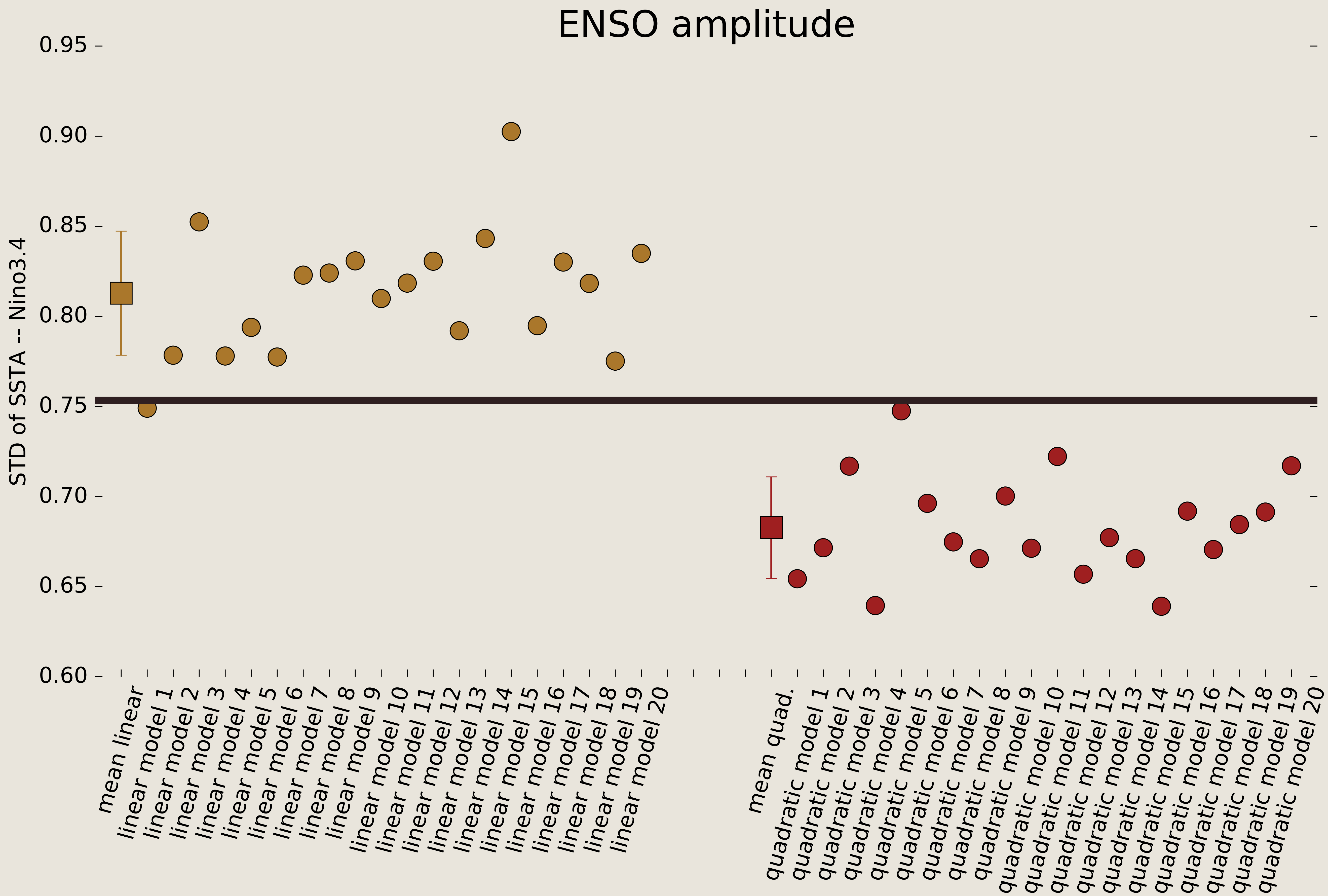
- model is estimated in the leading EOF space of Pacific sea surface temperature anomalies
- optimal number of state vector variables and degree of nonlinearity has to be assessed by cross-validation

Results basic ENSO metrics

- *NINO3.4* index - spatial average
- **amplitude** - STD of NINO3.4

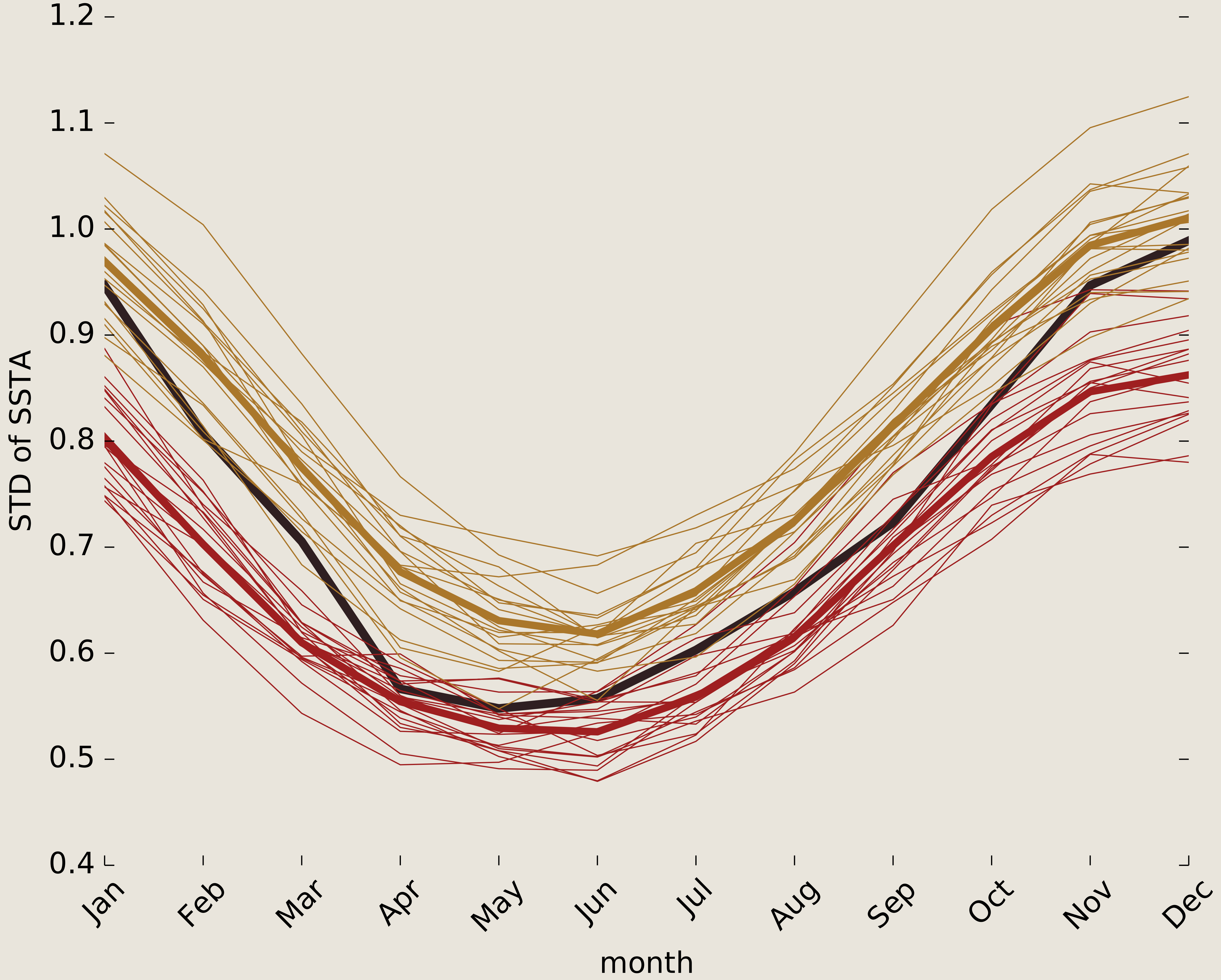


*noaa.gov



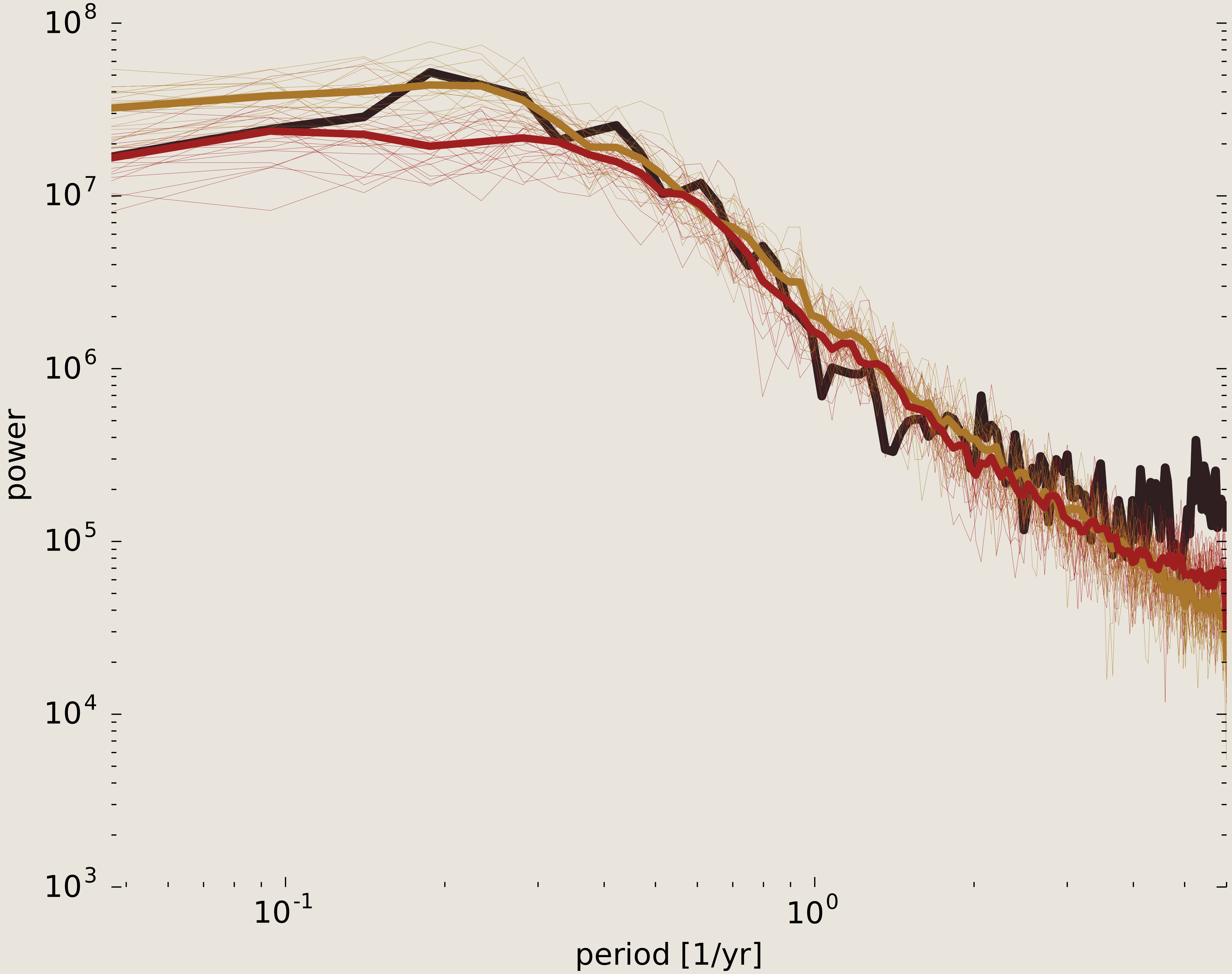
Results basic ENSO metrics

- **seasonality** - monthly STD of NINO3.4



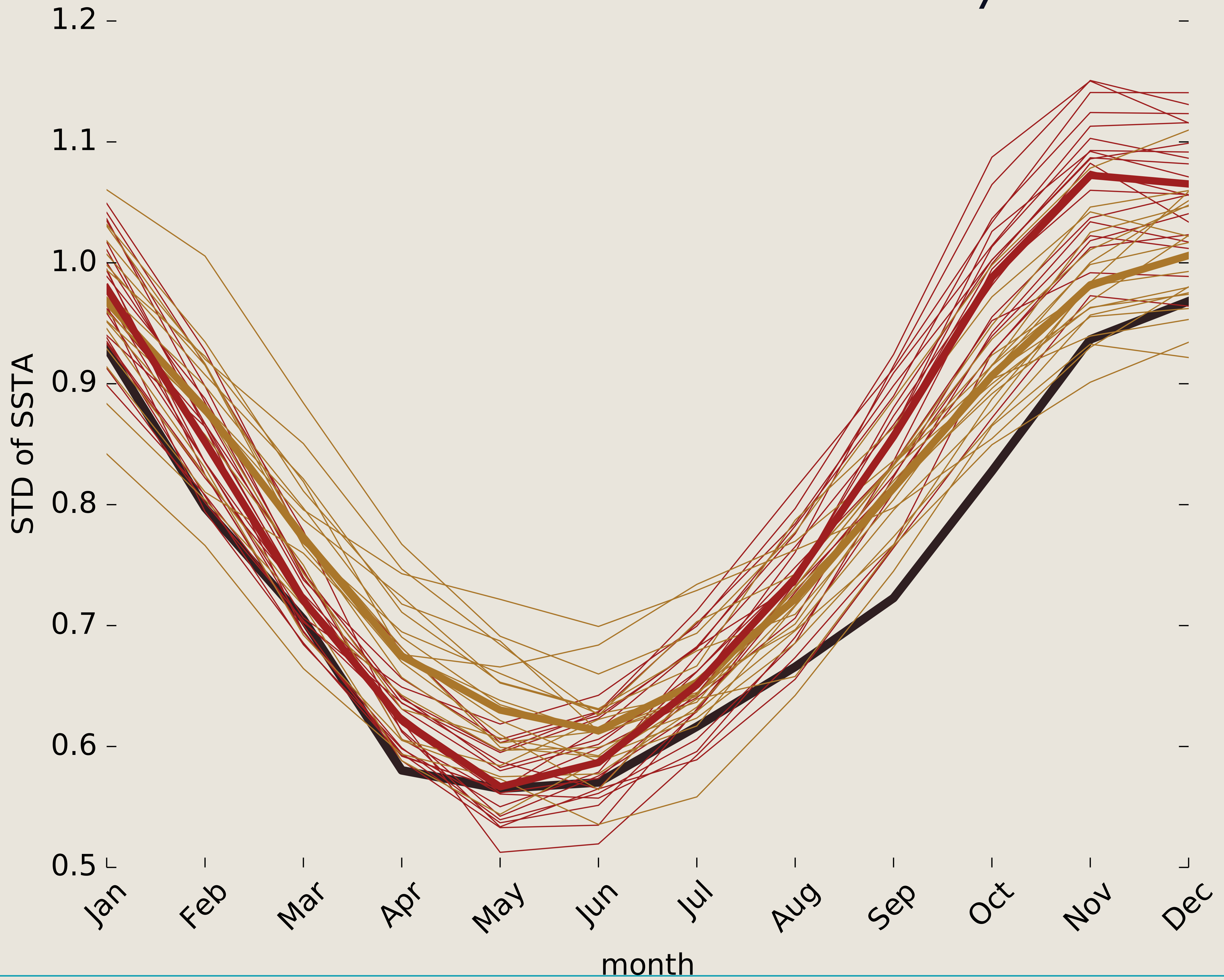
Results basic ENSO metrics

- **spectrum** - estimated using Welch method



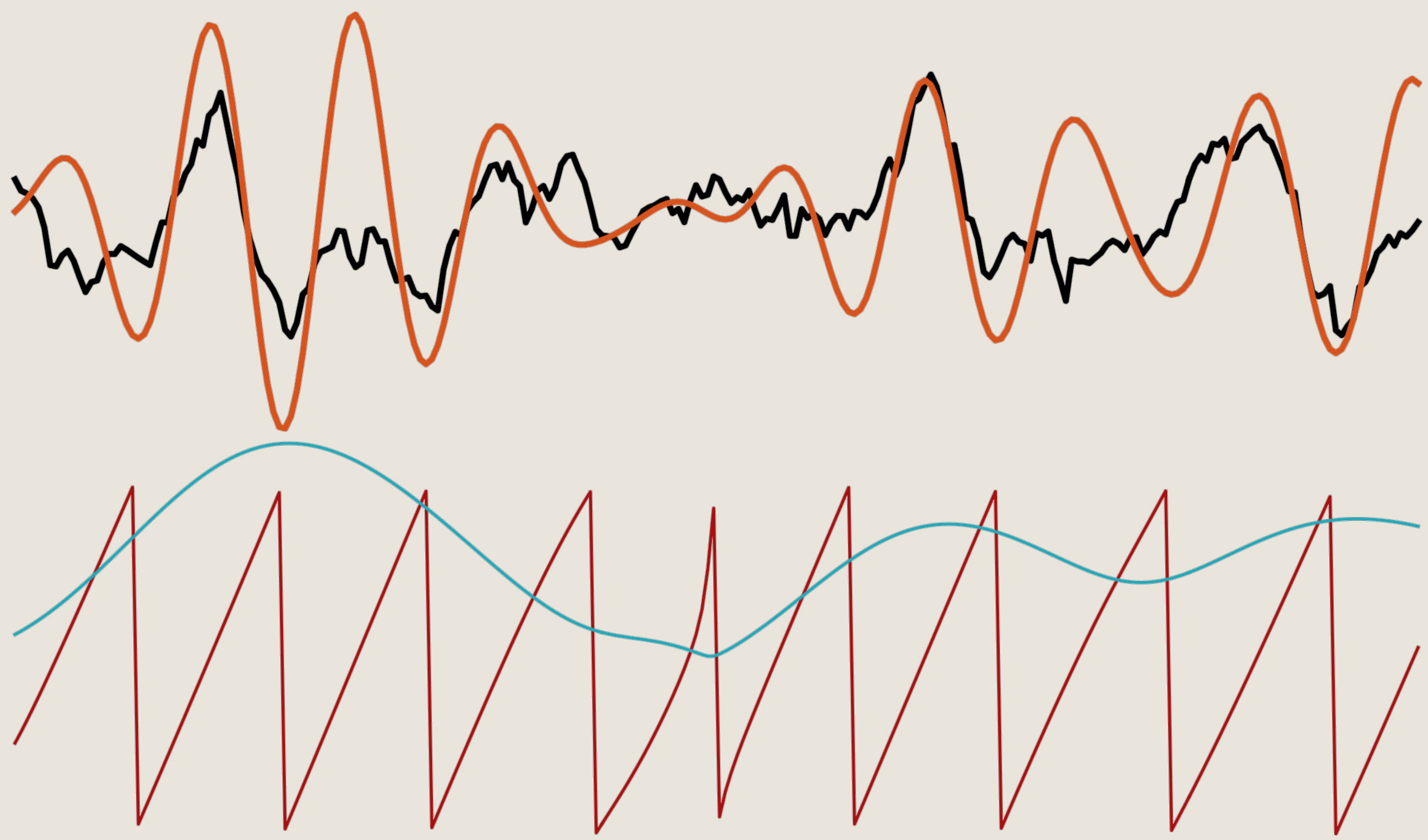
Noise parametrization seasonality

- even multi-level model exhibit serial correlations and seasonal dependence
- noise is conditioned on system's state



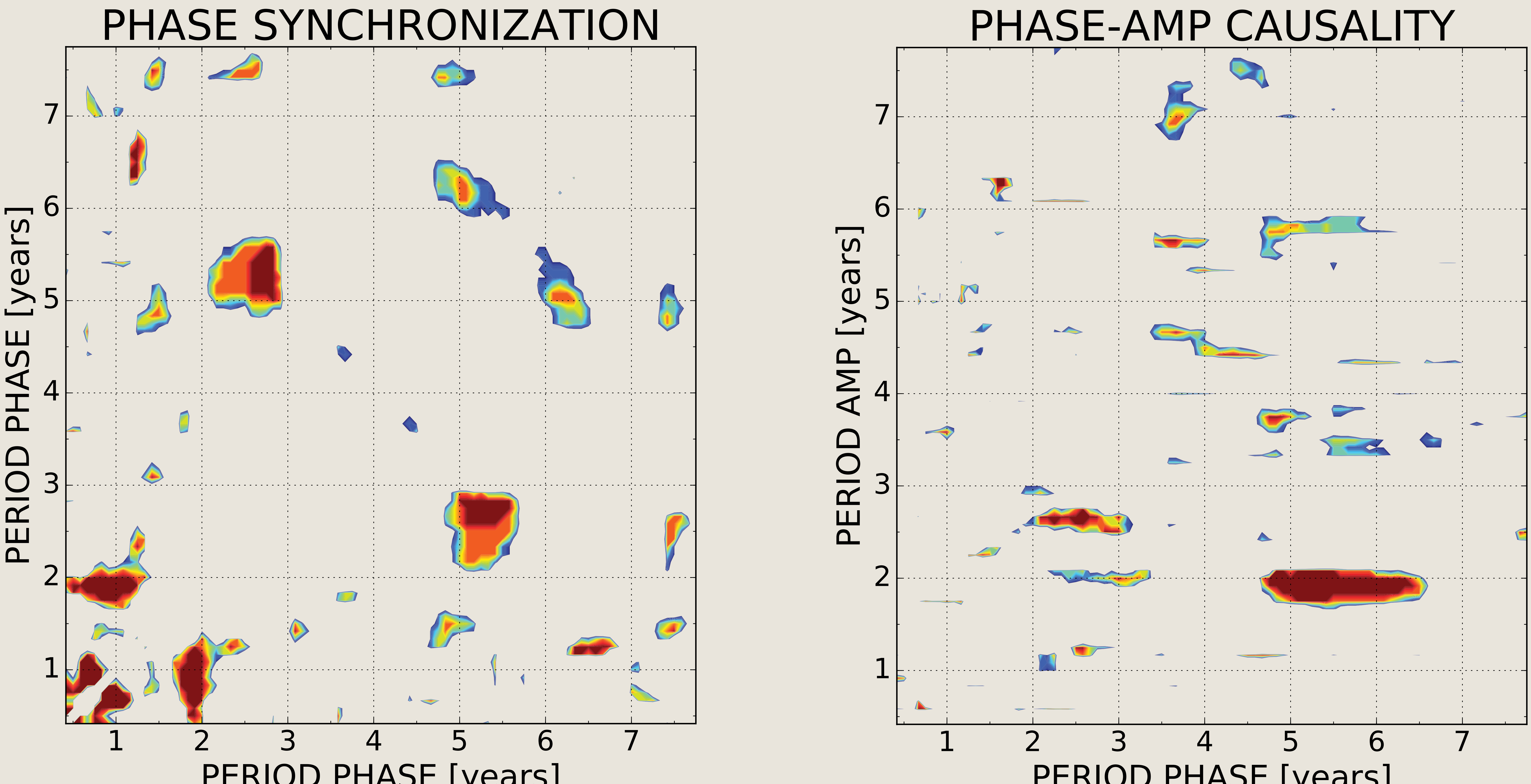
Synchronization and causality concept

- causal relations or information flow between various scales in the same variable / process
- using wavelet transform to infer instantaneous phase and amplitude of the signal with selected period



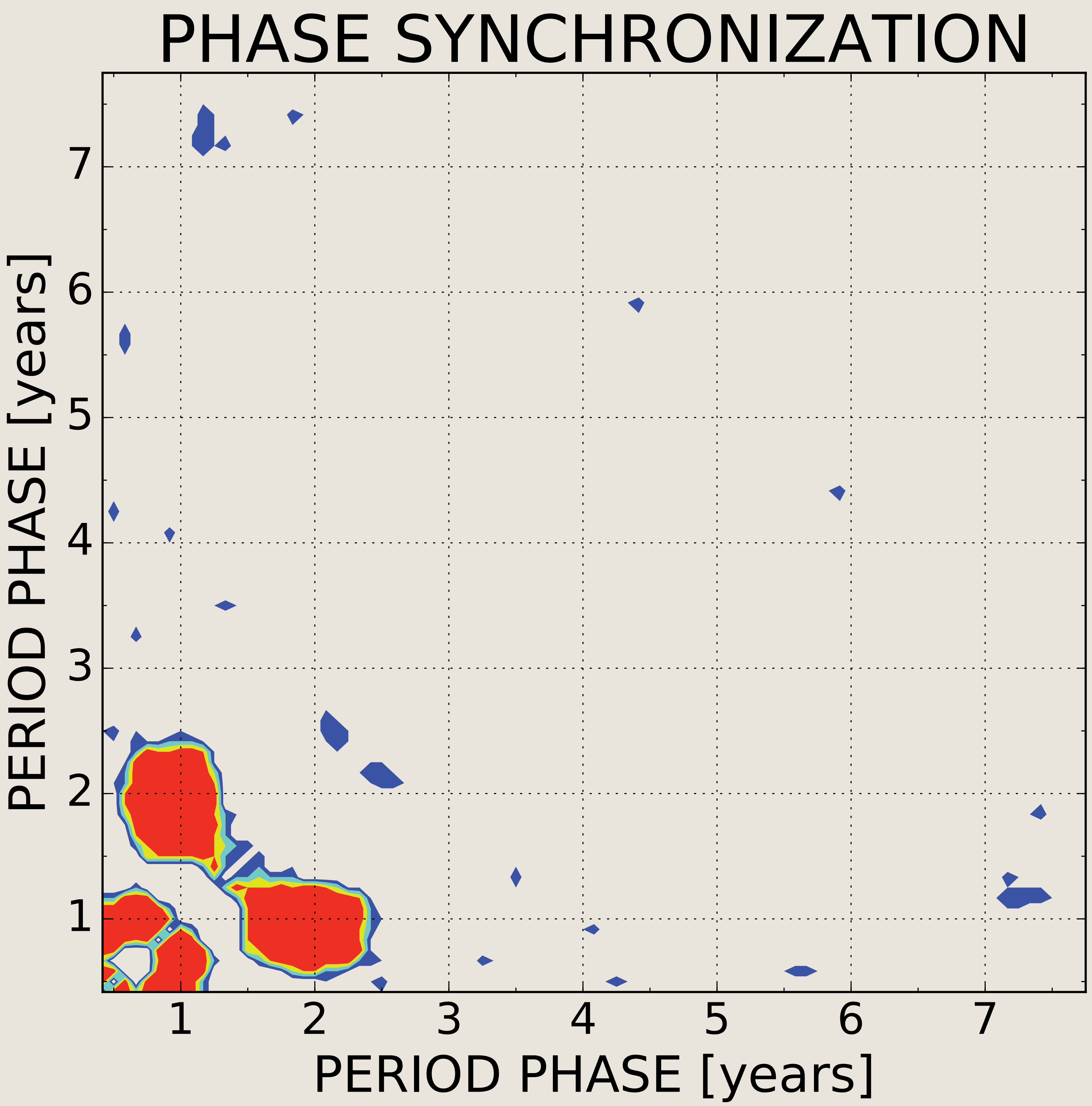
Synchronization and causality data

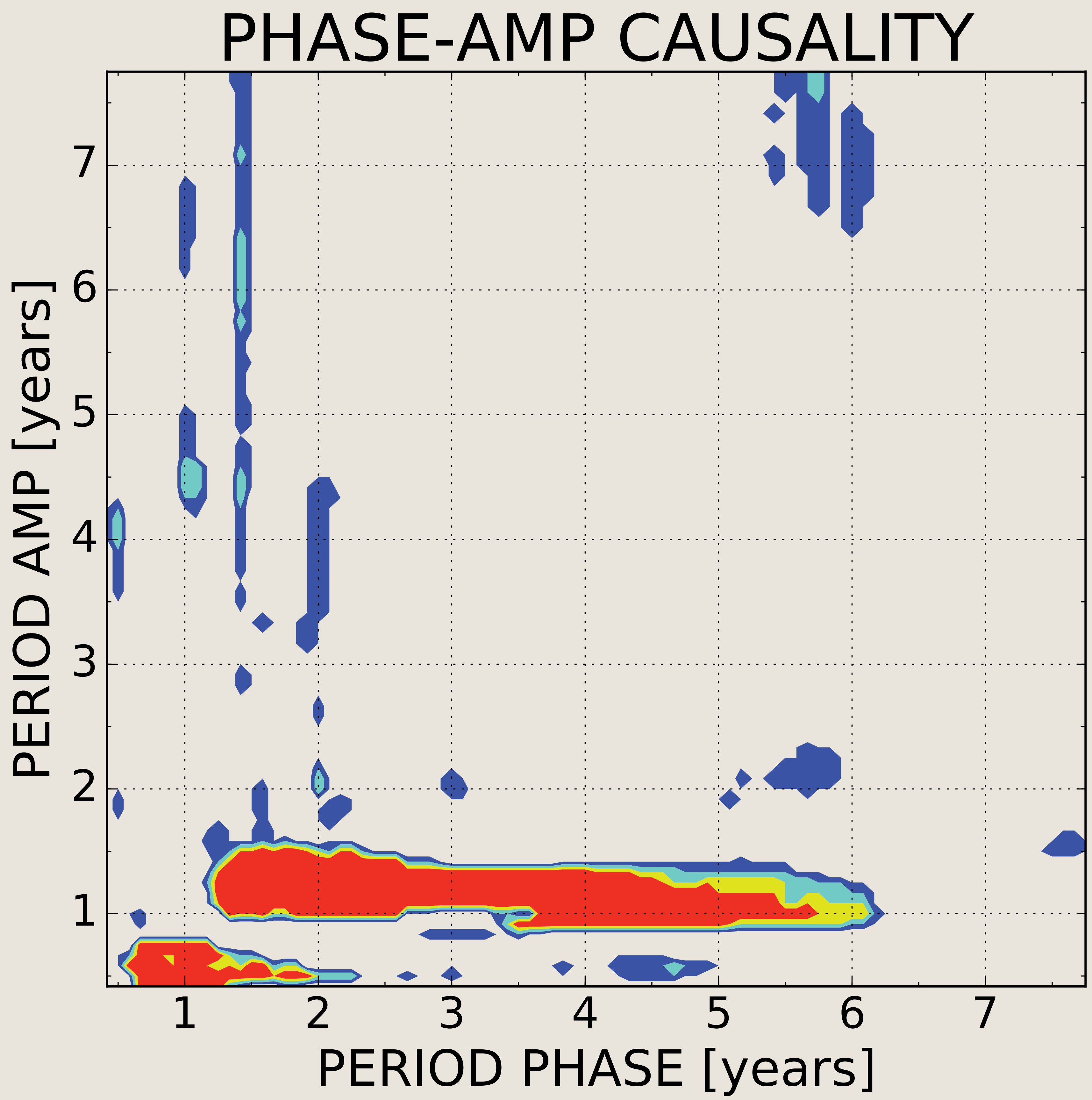
- using (conditional) mutual information to infer synchronization and causality measures



Synchronization and causality model

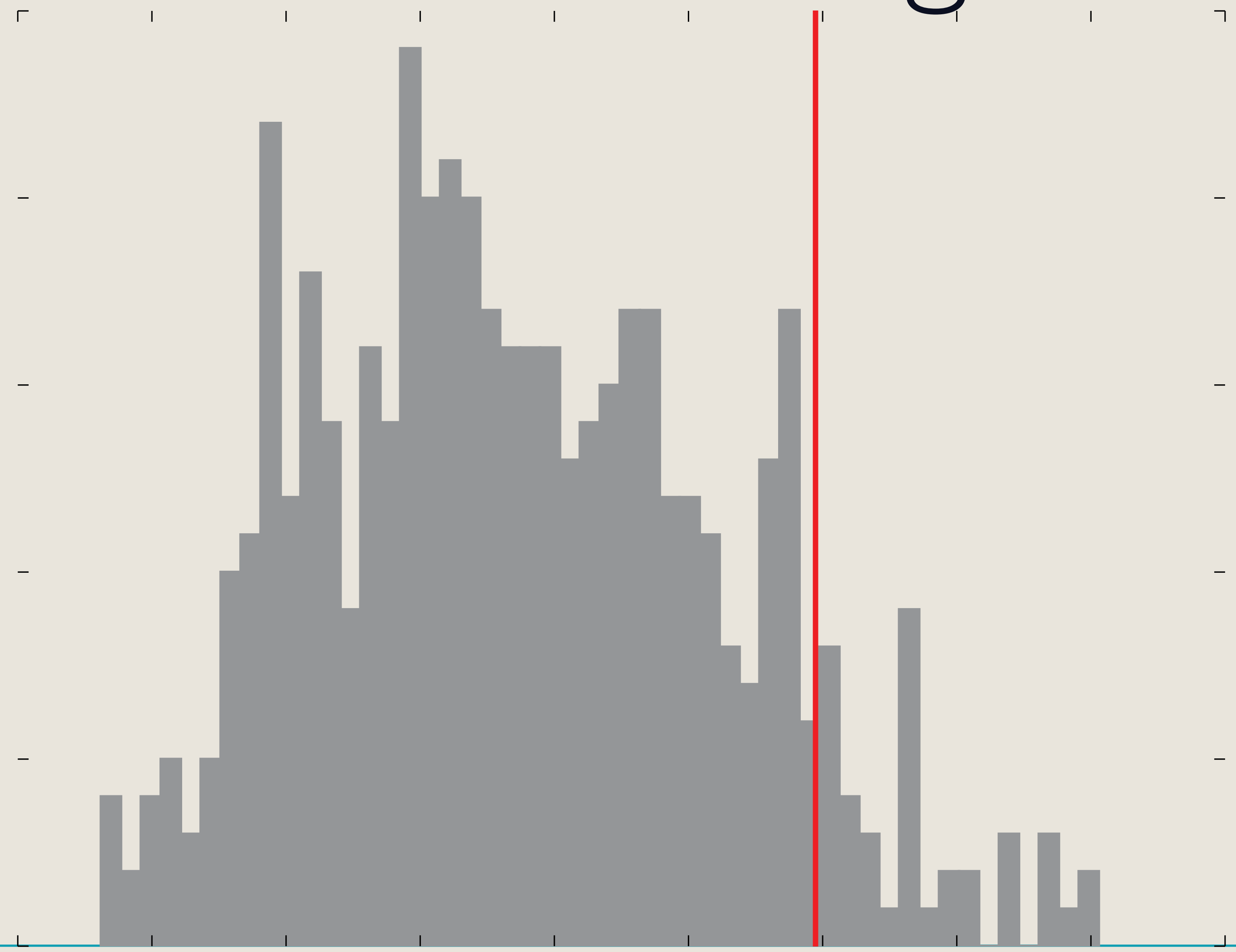
- simulate synchronization and causality in modelled time series to uncover the mechanisms





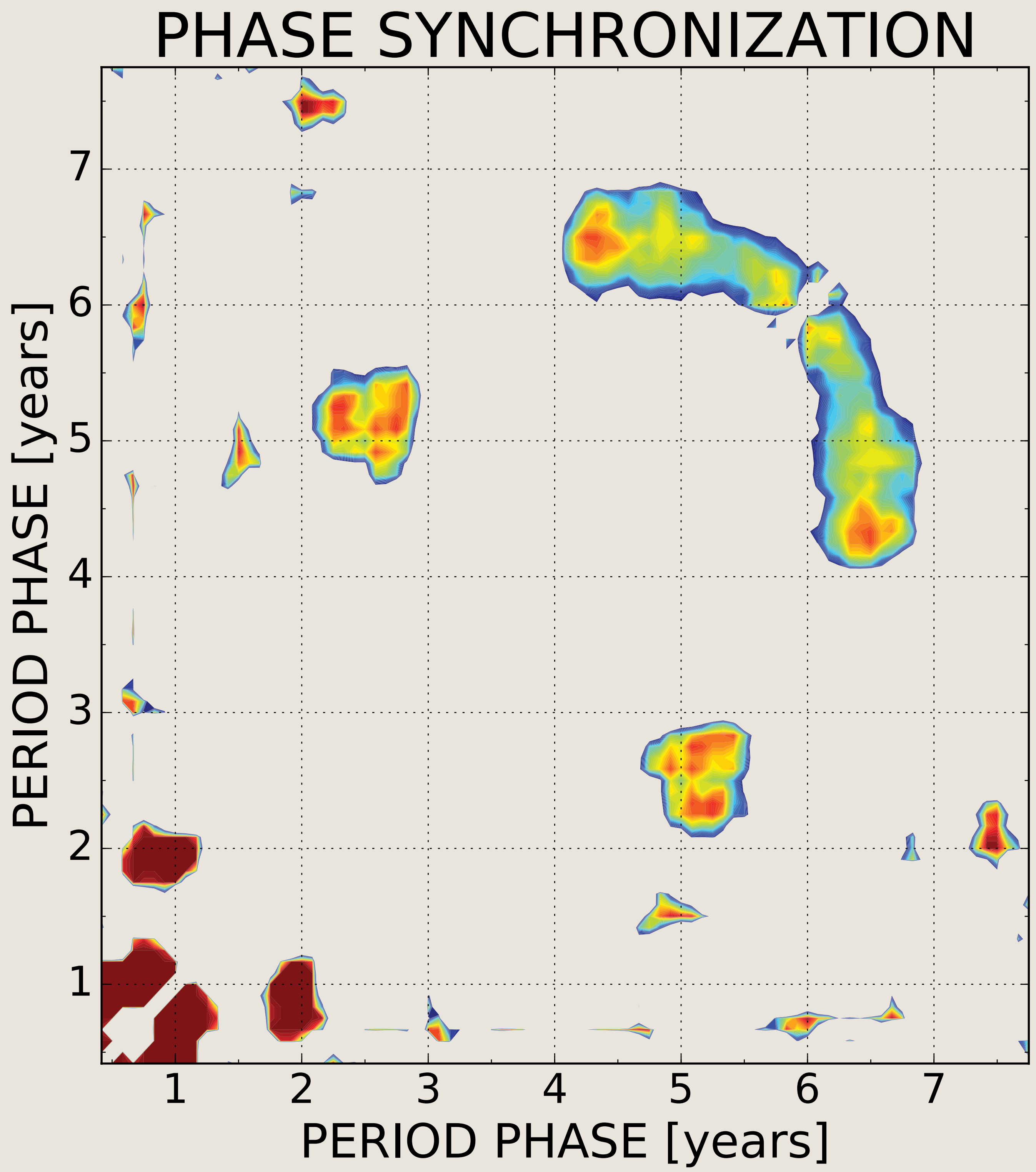
Surrogate data modelling with statistical model

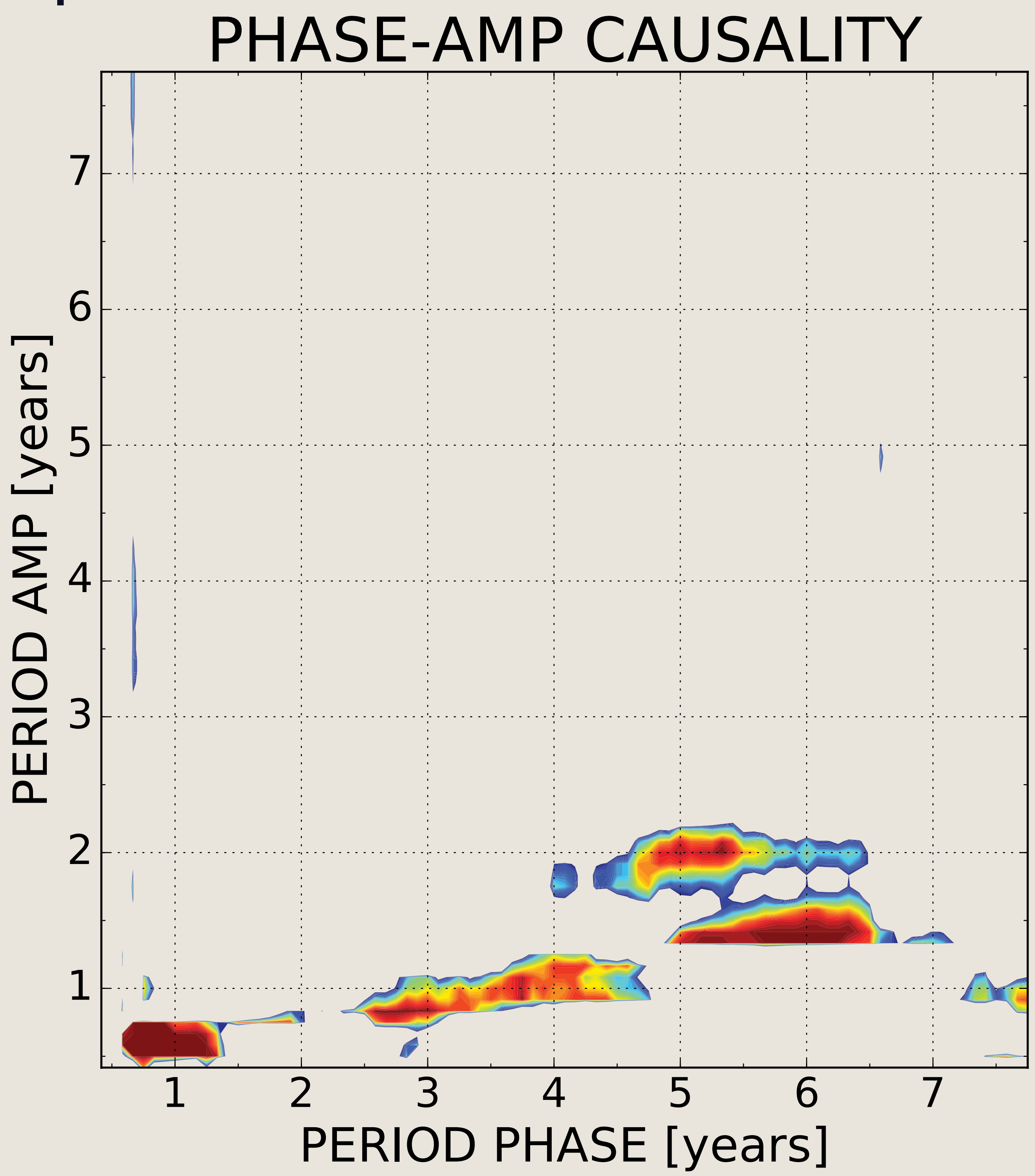
- method to generate synthetic that preserve some of the properties of the original data, while omitting the others
- use to test statistical significance by contradiction
- pose a null hypothesis and then generate an ensemble of surrogate time series using MC methods



Surrogate data modelling with statistical model

- more sophisticated null hypothesis: exploit the options of data-based model -- create surrogate ensemble statistical model with low complexity
- our case: linear, no seasonal dependence, white noise





Conclusions and outlook

- statistical models for scaling down the complexity
- modelling linear and non-linear interactions
- various noise parametrizations
- possible usage as models for generating ensembles of surrogate data for statistical testing
- two paths: focusing on a model itself (various settings, multi variables, etc..) or connection with dynamical models (e.g. for parametrization of sub-grid phenomena etc)

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Thanks for your attention!

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