# Outlier Detection and Explanation with Random Forests

Graph mining and outlier detection meets Logic proof tutoring

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# Outline

- Resolution for theorem prooving
- Graph data – student solutions
- New features - generalized subgraph patterns
- Class outlier detection
- Finding anomalous graphs
- Explanation of outliers

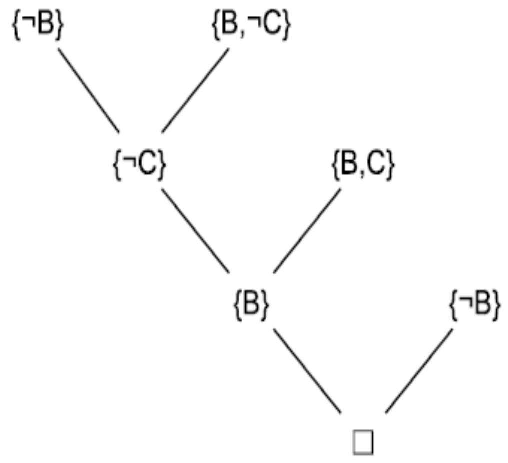
# Introduction

**Resolution in propositional logic**

- Theorem-proving technique based on refutation
- *Resolution rule* produces a new clause implied by two clauses
- *Example*: Prove that the following set of clauses is contradictory

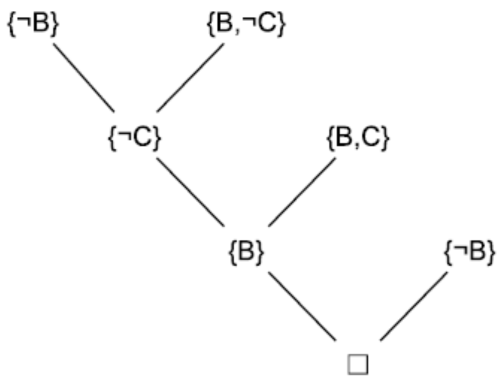
(B or ¬C) and ¬C and (B or C) and ¬B and (¬A,B) and (A or C)

{{B,¬C}, {¬C}, {B,C}, {¬B}, {¬A,B}, {A,C}}



# Method and goal

{{B,¬C}, {¬C}, {B,C}, {¬B), {¬A,B}, {A,C}}

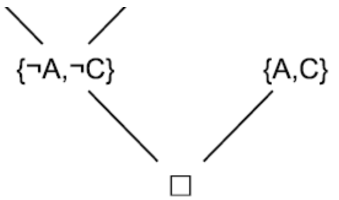



- graph mining, class outlier detection

- teaching enhancement by analysing students' solutions

# Data

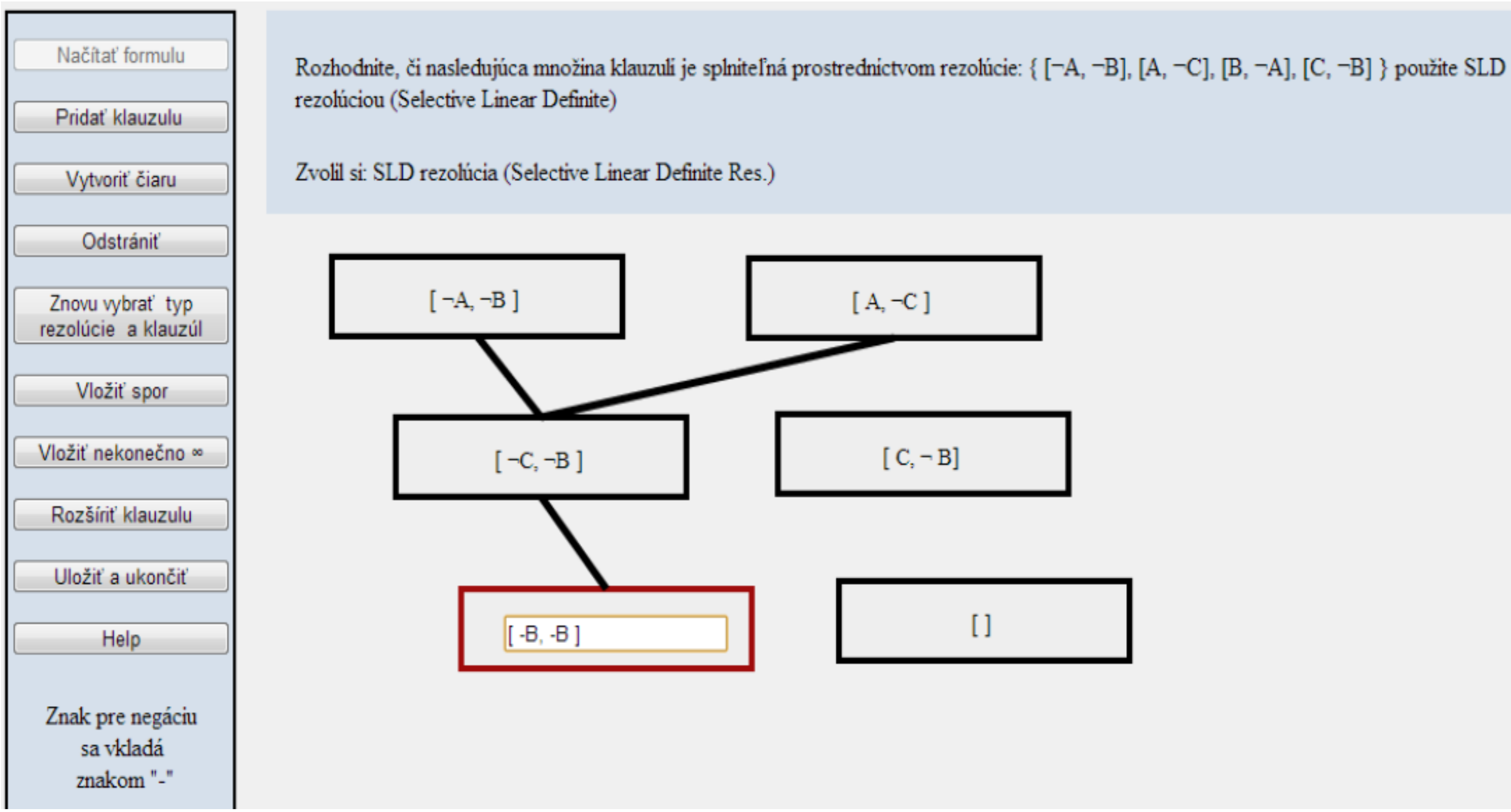
- 351 students solved resolution proofs via a web-based tool




- 873 resolution proofs classified into two classes
  - 772 correct
  - 101 incorrect

Incorrect ~ at least one error has been detected by an automatic evaluator

The most serious and common error: resolving on two literals at the same time


Načítať formulu
Pridať klauzulu
Vytvoriť čiaru
Odstrániť
Znovu vybrať typ rezolúcie a klauzúl
Vložiť spor
Vložiť nekonečno ∞
Rozšíriť klauzulu
Uložiť a ukončiť
Help
Znak pre negáciu sa vkladá znakom "-"
Rozhodnite, či nasledujúca množina klauzuli je splniteľná prostredníctvom rezolúcie: { [¬A, ¬B], [A, ¬C], [B, ¬A], [C, ¬B] } použite SLD rezolúciou (Selective Linear Definite)
Zvolil si: SLD rezolúcia (Selective Linear Definite Res.)
[ ¬A, ¬B ]
[ A, ¬C ]
[ ¬C, ¬B ]
[ C, ¬ B]
[ -B, -B ]
[ ]

# New features : subgraph patterns

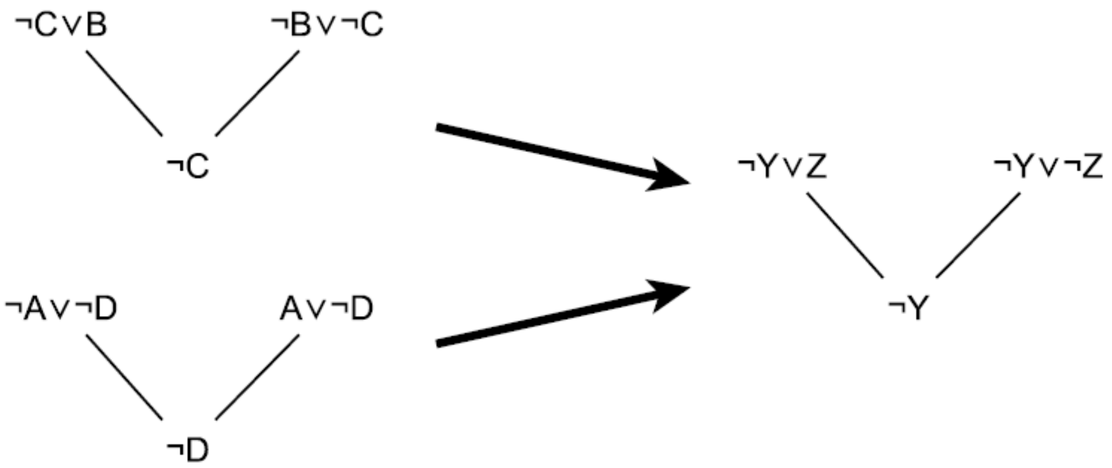
- Representation of graphs by their substructures – patterns
  - *Pattern$_i$* appeared in the graph -> true
  - *Pattern$_i$* did not appear in the graph -> false

| **pattern$_1$** | **pattern$_2$** | **...** | **pattern$_m$** | **class** |
|---|---|---|---|---|
| true | false | ... | false | incorrect |
| ... | ... | ... | ... | |
| false | true | ... | true | correct |

# Generalized Subgraph Patterns

- Representation of graphs by their substructures (patterns)
- Simple subgraphs inappropriate

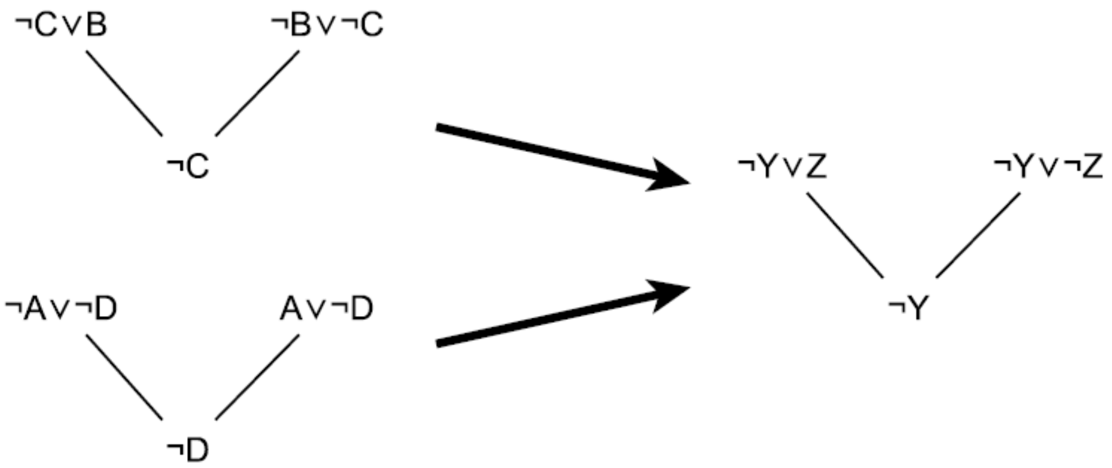
  => generalized subgraphs



# Generalized Subgraph Patterns

Procedure:

1. Extract all 3-node subgraphs (parents with the resolvent)
2. Perform generalization on these subgraphs



# Generalized Subgraph Patterns – Higher Level

- To increase performance of algorithms, we created a new, higher-level, generalization
- This method generalizes patterns created by previous gen. method
- It exploits domain knowledge about general resolution principle
- From 3-node generalized pattern it creates new pattern in following form:

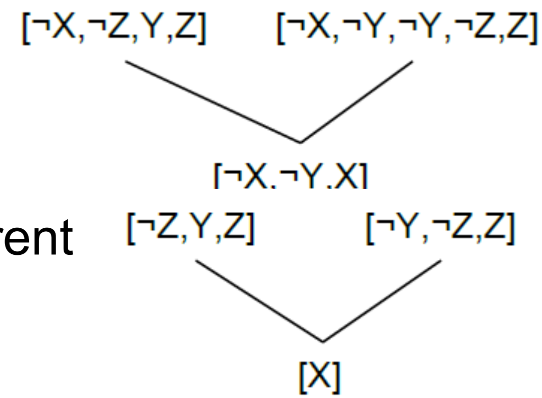
$$[L_{i_1}, L_{i_2}, \dots, L_{i_n}]; [L_{j_1}, L_{j_2}, \dots, L_{j_m}]$$

*(added)* (*dropped)*

- *added*: literals which were added erroneously to the resolvent
- *dropped*: literals from parents which participated in the resolution process

# Generalized Subgraph Patterns – Higher Level

Example:




- Go through literals in resolvent and delete those that occur in at least one parent

*added* = [X], *dropped* = [¬Z,Y,Z,¬Y,¬Z,Z]

- Rename letters in lists *dropped* and *added*, and sort both lists to get the final pattern

[Z];[¬X,¬X,¬Y,X,X,Y]

# Detection of anomalous solutions

(1a) Learn a classifier that would discriminate between correct and incorrect solutions, and

Detect rare cases that are incorrectly classified – outliers

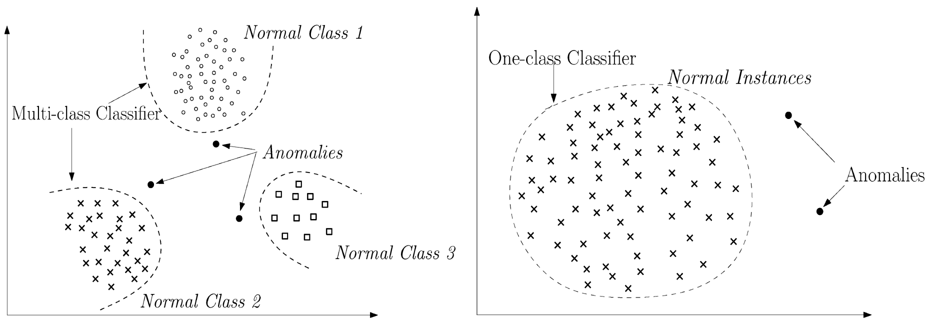
(1b) Clustering and observing class distribution in clusters

(2) BETTER: Directly detect outliers without learning a classifier.

Data have been classified => common outlier detection does not help

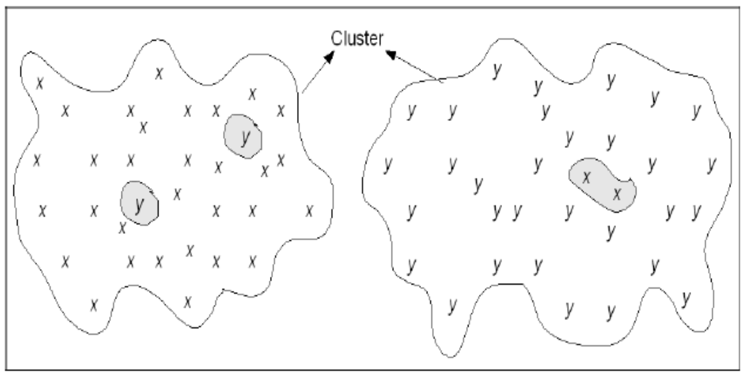
=> Class-based outlier detection

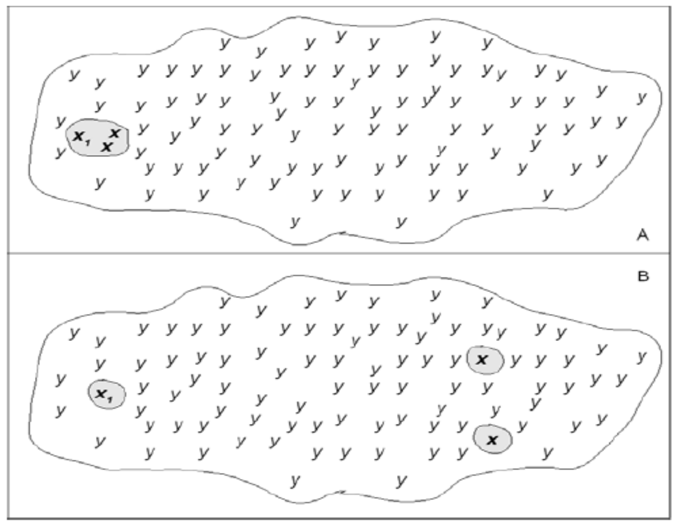
# Class outliers




(a) Multi-class Anomaly Detection

(b) One-class Anomaly Detection



# Class Outlier Detection

CODB (Hewahi and Saad 2007) – distance and density based

- weka-peka  (Pekarčíková 2013)
  - Based on Random Forests
  - Analysis of proximity matrix

# Class Outlier Detection via Random Forest

weka-peka (Pekarčíková 2013)

- Learn Random Forest
- *A*fter each tree is built, all of the data are run down the tree, and proximities are computed for each pair of cases:

  If two cases occupy the same terminal node, their proximity is increased by one.
- At the end of the run, the proximities are normalized by dividing by the number of trees.

# Class Outlier Detection via Random Forest

weka-peka (Pekarčíková 2013)

FO($p$) = FO1($p$) + FO2($p$) + cFO3($p$)

- 1/FO1($p$) … proximity of $p$ to elements from the same class
- FO2($p$) …… frequency of incorrect classification of $p$
- FO3($p$) ……. proximity $p$ to all elements, i.e. when ignoring the class attribute

# Outlier detection by weka-peka

| Instance | Error E3 | Outlier score |
|---|---|---|
| 270 | no | 131.96 |
| 396 | no | 131.96 |
| 236 | no | 73.17 |
| 187 | no | 61.03 |
| 438 | yes | 54.43 |
| 389 | yes | 52.50 |
| 74 | yes | 15.91 |
| 718 | yes | 15.91 |

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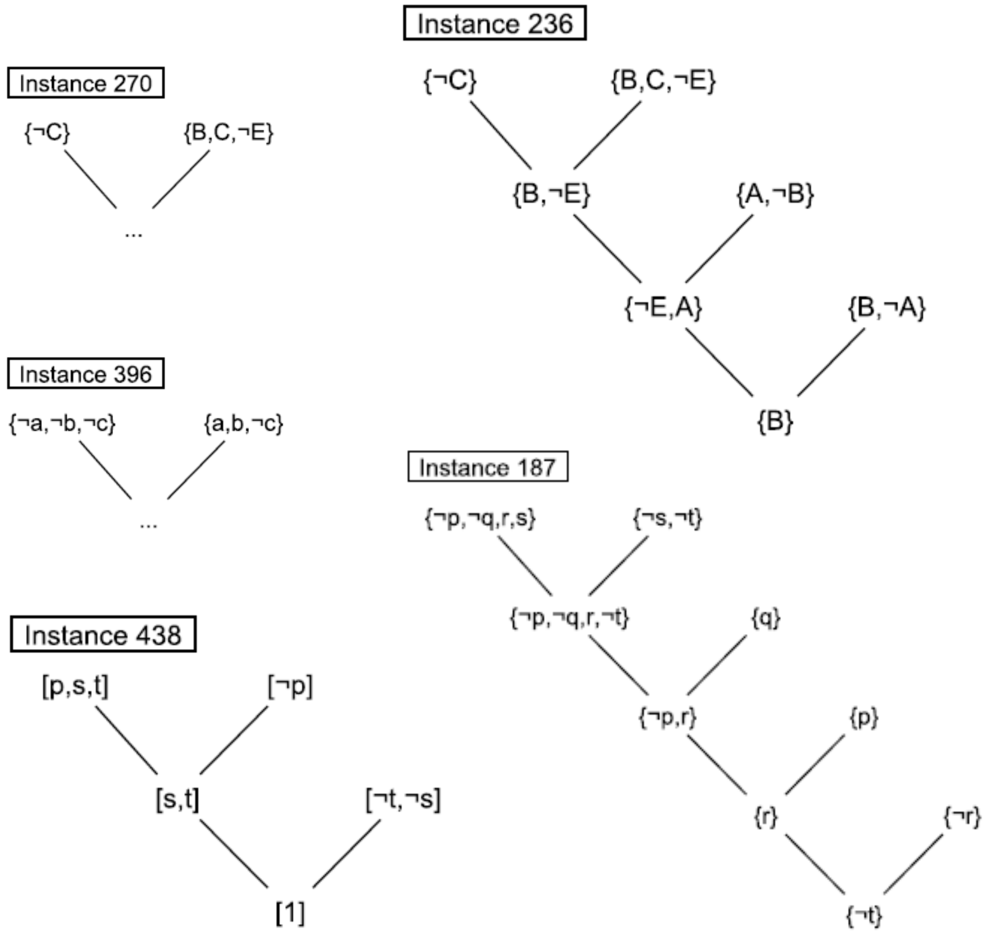

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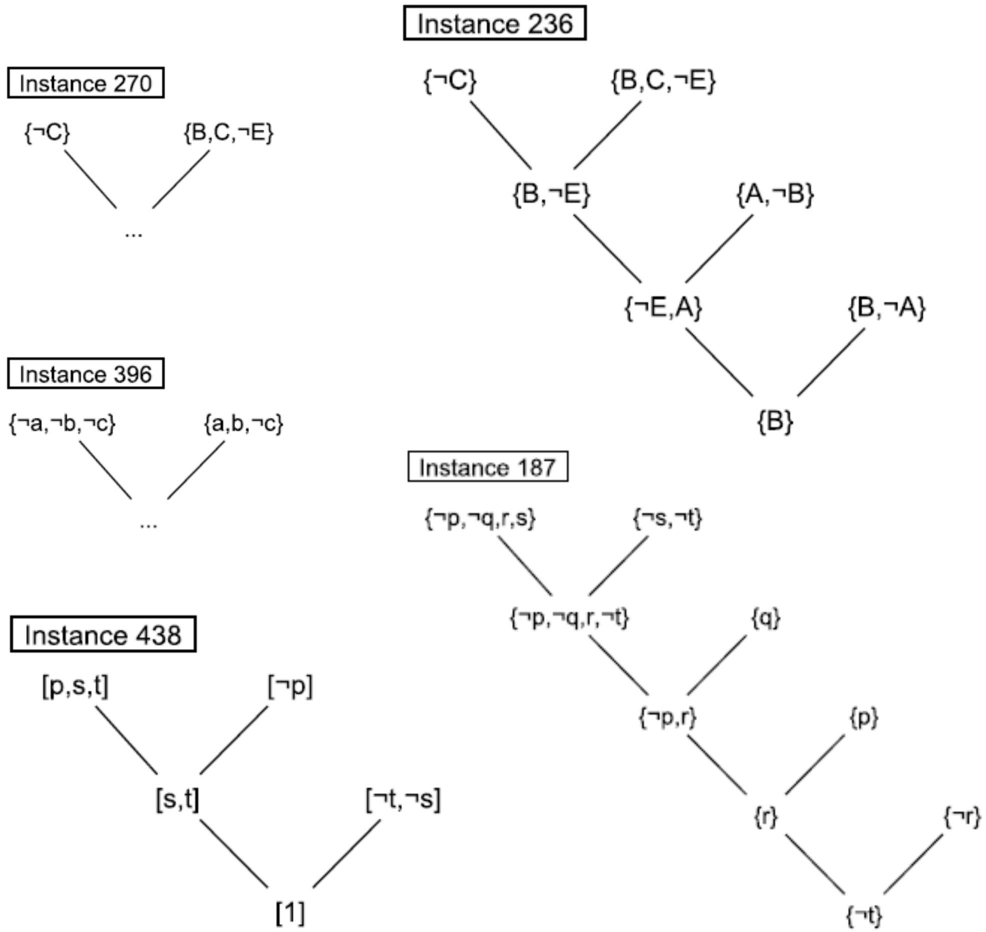



Goal now: to explain/interpret outliers

# References

VACULÍK, Karel, NEZVALOVÁ, Leona a Lubomír POPELÍNSKÝ. Graph Mining and Outlier Detection Meet Logic Proof Tutoring. Proc. of EDM 2014 Ws Graph-based Educational Data Mining

VACULÍK, Karel, Leona NEZVALOVÁ a Lubomír POPELÍNSKÝ. Educational data mining for analysis of students' solutions.16th International Conference, AIMSA 2014. London: Springer, 2014.

CODB:

Hewahi N.M. and Saad M.K. *Class Outliers Mining: Distance-Based Approach*. International Journal of Intelligent Systems and Technologies, Vol. 2, No. 1, pp 55-68, 2007.

pattern text

1 {} ; {neg Z,Z}

2 {} ; {}

3 {} ; {neg Z}

4 ...(cykli) ; ...(cykli)

5 {neg Z} ; {neg Y,Y}

6 {neg Y,Z} ; {neg X,X}

7 {} ; {Z}

8 {} ; {neg Y,neg Z,Y,Z}

9 {Y} ; {neg Y,neg Z,Z}

10 {} ; {Z,Z}

11 {neg Y} ; {neg Z,Y,Z}

12 {} ; {neg Z,neg Z}

13 {Z} ; {neg Y,Y}

14 {} ; {neg Y,neg Z,Y}

15 {} ; {neg Y,Y,Z,Z}

16 {Z} ; {neg X,neg Y,X,Y}

17 {neg Z} ; {neg X,neg Y,X,Y}

18 {} ; {neg X,neg Y,neg Z,X,Y,Z}

19 {Y} ; {neg X,X,Z}

20 {} ; {neg Y,Z}

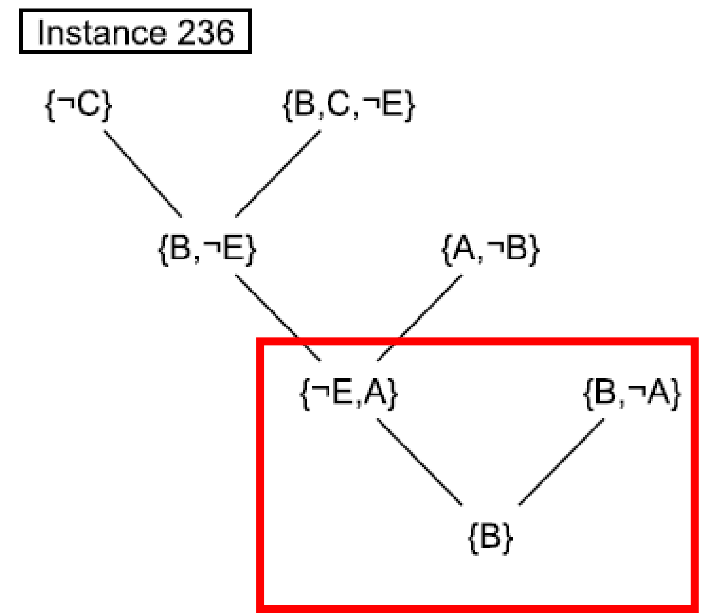
# AScore

- Let *P* be a pattern and suppose its value is *true* for the outlier
- Compute AScore of *P* as the fraction of remaining instances from the same class as the outlier whose value of *P* is equal to *false*
- Similarly for *P* equal to *false*, but AScore ranges from 0 to -1

# Outlier Explanation

| Instance | Error E3 | Outlier score | Significant patterns* [(Ascore) *added; dropped*] | Significant missing patterns* [(Ascore) *added; dropped*] |
|---|---|---|---|---|
| 270 | no | 131.96 | (0.96) *looping* | (-0.99) {};{¬Z,Z} |
| 396 | no | 131.96 | (0.96) *looping* | (-0.99) {};{¬Z,Z} |
| 236 | no | 73.17 | (0.99) {};{¬Y, ¬Z,Y} | |
| 187 | no | 61.03 | (0.99) {¬Z};{¬Y,Y}<br>(0.99) {};{¬Y, ¬Z,Y} | |
| 438 | yes | 54.43 | (1.00) {Z};{¬X, ¬Y,X,Y} | (-0.94) {};{¬Y, ¬Z,Y,Z} |
| 389 | yes | 52.50 | (1.00) {};{¬Y, ¬Z,Y} | (-0.94) {};{¬Y, ¬Z,Y,Z}<br>(-0.81) {};{¬Z,Z} |
| 74 | yes | 15.91 | (0.98) {¬Z};{¬X, ¬Y,X,Y}<br>(0.98) {};{¬X, ¬Y, ¬Z,X,Y,Z} | (-0.94) {};{¬Y, ¬Z,Y,Z} |
| 718 | yes | 15.91 | (0.98) {¬Z};{¬X, ¬Y,X,Y}<br>(0.98) {};{¬X, ¬Y, ¬Z,X,Y,Z} | (-0.94) {};{¬Y, ¬Z,Y,Z} |

*Only patterns with |AScore| > 0.8 are displayed



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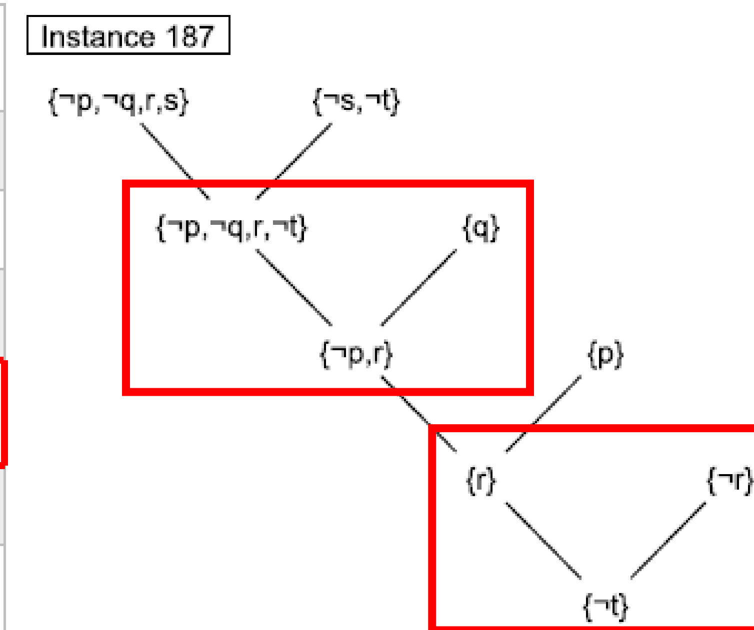


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| 438 | yes | 54.43 | (1.00) {Z};{¬X, ¬Y,X,Y} | (-0.94) {};{¬Y, ¬Z,Y,Z} |
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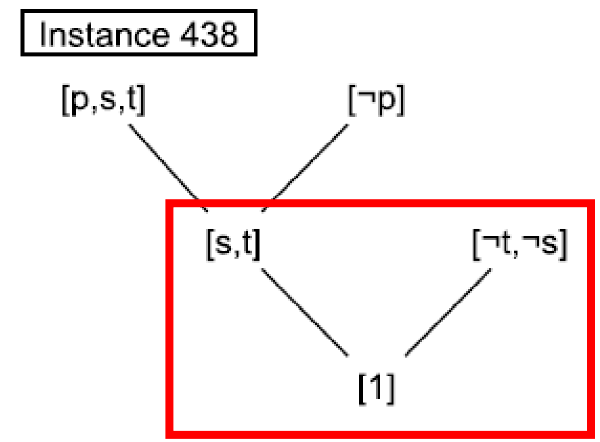
| 718 | yes | 15.91 | (0.98) {¬Z};{¬X, ¬Y,X,Y}<br>(0.98) {};{¬X, ¬Y, ¬Z,X,Y,Z} | (-0.94) {};{¬Y, ¬Z,Y,Z} |

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| 236 | no | 73.17 | (0.99) {};{¬Y, ¬Z,Y} | |
| 187 | no | 61.03 | (0.99) {¬Z};{¬Y,Y}<br>(0.99) {};{¬Y, ¬Z,Y} | |
| 438 | yes | 54.43 | (1.00) {Z};{¬X, ¬Y,X,Y} | (-0.94) {};{¬Y, ¬Z,Y,Z} |
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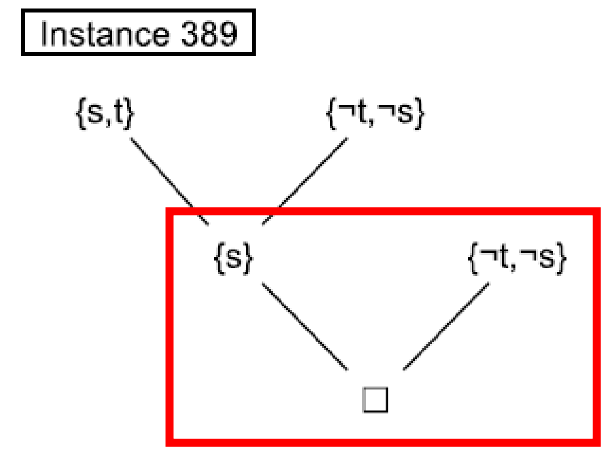

# Sapling Random Forest (LOF)

- Train trees (sapling) to distinguish normal instances from outliers
- Only subset of normal instances are used (k-nereast neighbours vs. random)
- Find explanation in resulting paths
- Knowledge about classes not taken into account

# Tree Reduction

- Original classes replaced with two new classes
  - Normal class – includes labeled class of given instance
  - Outlier class – includes others original classes
- Analysis of the trees that classify instance to Outlier class
- Change attribut values on the path in given tree for given instance and check if resulting class change → attribut is part of explanation
- Find explanation in resulting paths and calculate support

# Results

| Instance | Error E3 | Outlier score | AScore | Sapling Random Forest (LOF) | Tree Reduction |
|---|---|---|---|---|---|
| 270 | no | 131.96 | pattern_1 (-0.99)<br>pattern_4 (0.96) | pattern_1=0 (1) | pattern_1=0 (0.81) |
| 396 | no | 131.96 | pattern_1 (-0.99)<br>pattern_4 (0.96) | pattern_1=0 (1) | pattern_1=0 (0.81) |
| 236 | no | 73.17 | pattern_14 (0.99) | pattern_2=0 (0.8) | pattern_14=1 (0.46)<br>pattern_14=1 && pattern_5=0 (0.22) |
| 187 | no | 61.03 | pattern_14 (0.99)<br>pattern_5 (0.99) | pattern_2=0 (0.95) | pattern_14=1 (0.76) |
| 438 | yes | 54.43 | pattern_16 (1.00)<br>pattern_8 (-0.94) | pattern_2=0 (0.75) | pattern_1=1 && pattern_8=0 && pattern_17=0 (0.23)<br>pattern_8=0 && pattern_17=0 (0.20) |
| 389 | yes | 52.50 | pattern_14 (1.00)<br>pattern_8 (-0.94)<br>pattern_1 (-0.81) | pattern_1=0 (1) | pattern_8=0 (0.26)<br>pattern_8=0 && pattern_16=0 && pattern_17=0 (0.21)<br>pattern_8=0 && pattern_16=0 && pattern_18=0 (0.21) |
| 74 | yes | 15.91 | pattern_17 (0.98)<br>pattern_18 (0.98)<br>pattern_8 (-0.94) | pattern_2=0 (0.7) | pattern_8=0 && pattern_16=0 (0.29)<br>pattern_1=1 && pattern_8=0 && pattern_16=0 (0.26) |