Optimization in Smart Grids

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Smart Grids

Smart grids

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Demand side management

Demand side management (DSM) is the modification of consumer demand for energy through various methods such as financial incentives and behavioral change through education. Usually, the goal of demand side management is to encourage the consumer to use less energy during peak hours, or to move the time of energy use to off-peak times such as nighttime and weekends.

Micro-grids

Motivations to balance electricity locally

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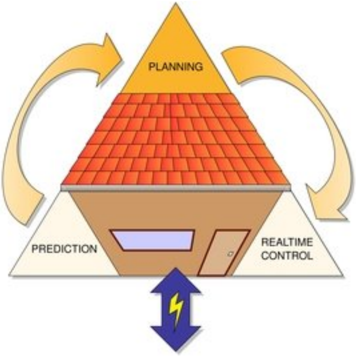
- Reduce electricity losses on a distribution network
- Reduce investments on a distribution network

Contents

Triana

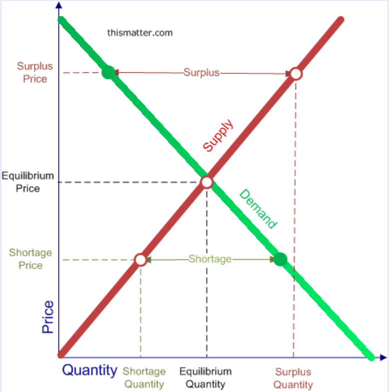
Steps

1. Prediction
2. Planning
3. Real-time control



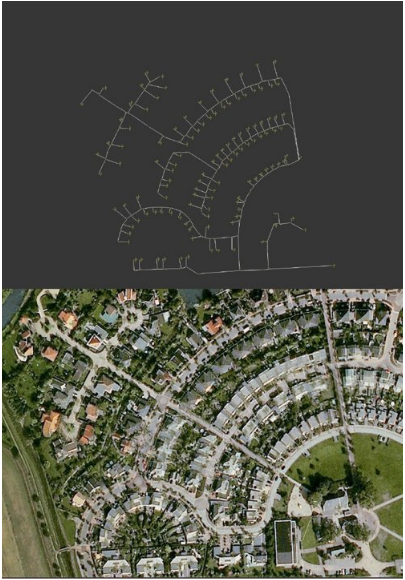
Approach

PowerMatcher is based on the market equilibrium.



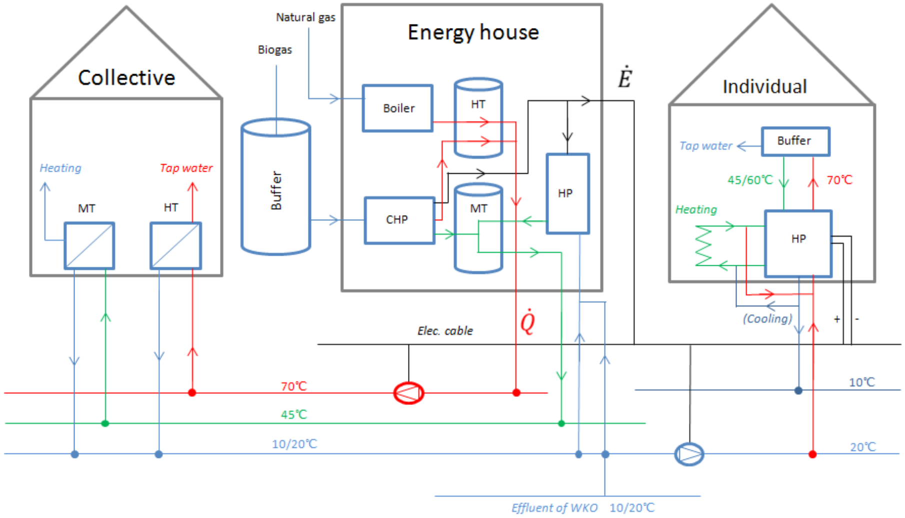
Contents

Lochem: Village in the Netherlands



Meppel: Town in the Netherlands

Project MeppelEnergie



Project background

Concept

- Housing project Meppel
- District heating with biogas CHP
- Domestic heat pumps
- Smart control: CHP and Heat pumps

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Research objectives

- Reduce aggregated peak electric loads caused by domestic heat pumps
- Use as much of the CHP-generated electricity as possible for the heat pump demand

Contents

Model for simulation

Key questions

- To what extend is it possible to reduce peak electricity demand of all heat pumps?
- How to maintain thermal comfort within the houses if the heat pump control is not based on conventional thermostat control?
- Is the obtained heat pump control sensible?
- Is the control method able to increase self consumption of generated electricity?

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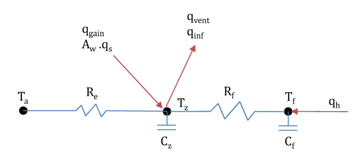
Approach

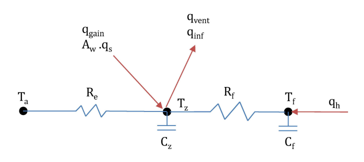
- Simulation study of 104 houses with a heat pump
- Incorporate demand for space heating including:
  - Building heat loss
  - Ventilation heat loss
  - Solar heat gains through windows
  - Internal gains due to people and appliances
- Incorporate domestic hot water demand schedules
- Week with high + week with low space heating demand
- Simulate individual “reference control” for all houses
- Develop central optimal control method and simulate all houses

The model



Thermo-resistance model

The model



Heat demand calculation

$$\frac{C_f}{\tau}(T_{f,t+1} - T_{f,t}) \quad = \quad \frac{T_{z,t} - T_{f,t}}{R_f} + q_{h,t}$$

Thermo-resistance model

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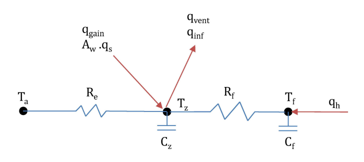

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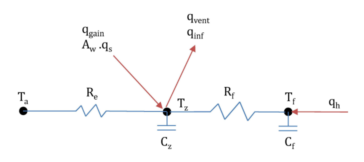

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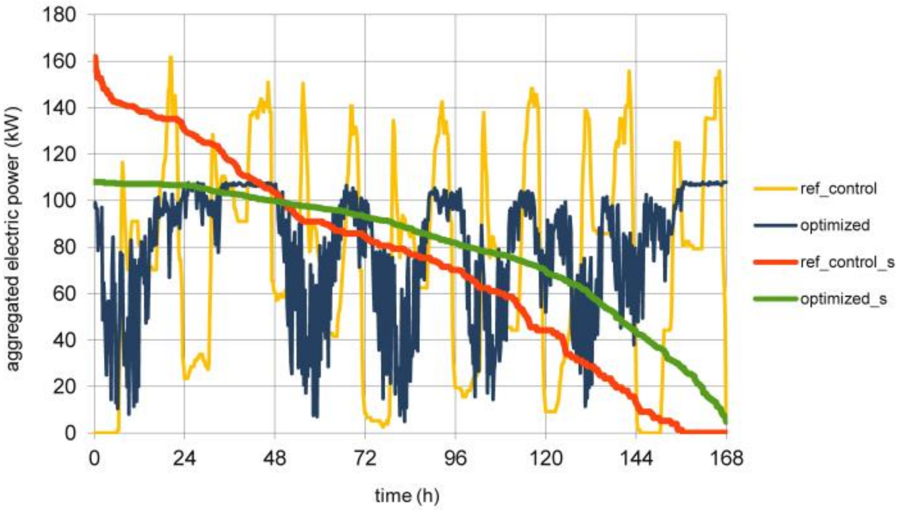
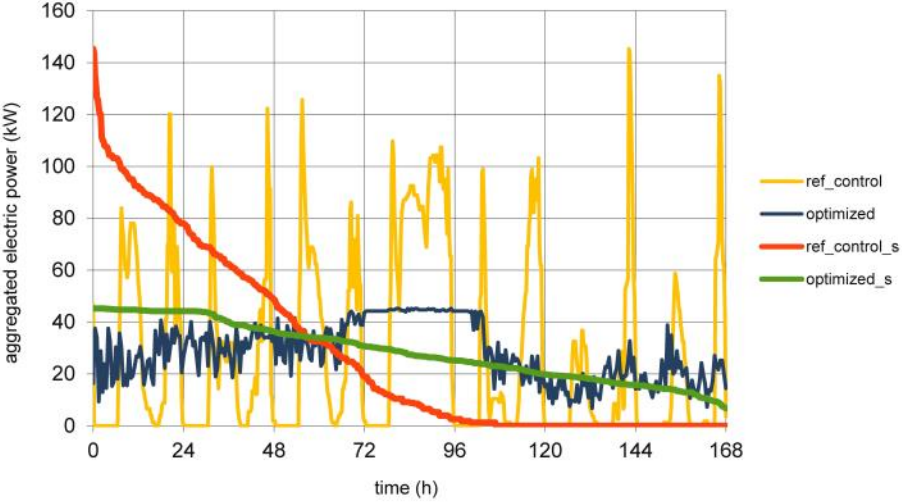
$$\text{Minimize } \alpha \sum_t \max\left\{T_{pref,t} - T_{z,t}, 0\right\} + \beta \sum_t q_{h,t}$$

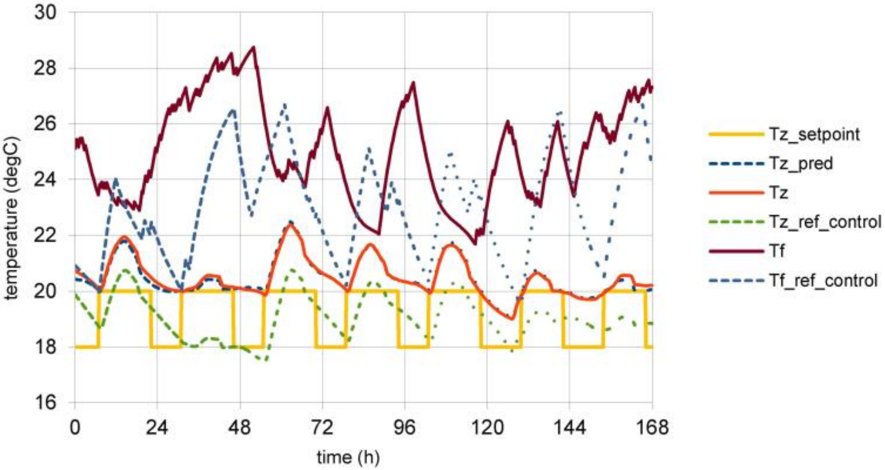
Aggregated electricity: A week in winter



Aggregated electricity: A week in summer



Aggregated electricity: Indoor temperature in winter



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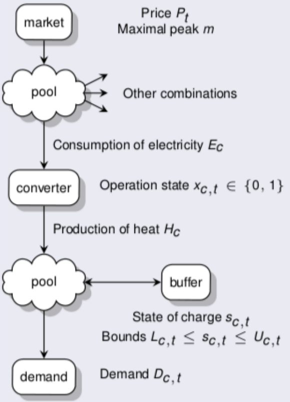
Problem description

Schema



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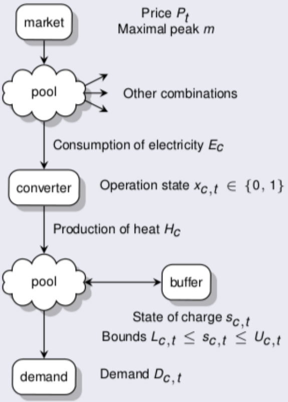

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$$s_{c,t+1} = s_{c,t} + H_c x_{c,t} - D_{c,t}$$

$$L_{c,t} \leq s_{c,t} \leq U_{c,t}$$

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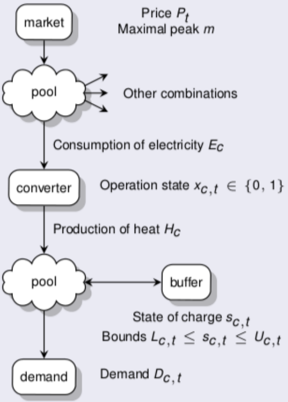

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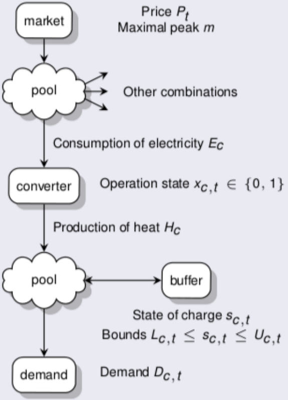

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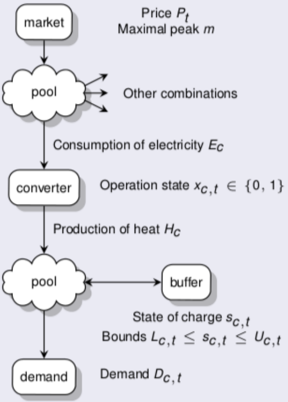

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Applications

- Heating water
- House heating
- Fridges and freezers
- Energy production

Peak shaving

Find scheduling of converters fulfilling demands which minimize the maximal peak.

Algorithms and complexity

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- Dynamic programming: $\mathcal{O}(T^2)$
- Greedy algorithm: $\mathcal{O}(T^2)$
- Greedy algorithm with union-find: $\mathcal{O}(T\alpha(T))$ + sorting

Replace the recurrence formula by an explicit formula

$$s_{c,t+1} = s_{c,t} + H_c x_{c,t} - D_{c,t}$$

Preprocessing

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Bounds on the state of charge

$$L_{c,t+1} \le s_{c,t+1} \le U_{c,t+1}$$

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where $A_{c,t}$ and $B_{c,t}$ are integers and
$A_{c,t} \le A_{c,t+1} \le A_{c,t} + 1$ and $B_{c,t} \le B_{c,t+1} \le B_{c,t} + 1$ and $A_1, B_1 \in \{0, 1\}$.

Preprocessing

Conditions on variables $x_{c,t}$

- $x_{c,t} \in \{0, 1\}$
- $A_{c,t} \leq \sum_{i \leq t} x_{c,i} \leq B_{c,t}$

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Objective functions

Minimizing cost: $\text{minimize} \sum_t \sum_c P_t E_c x_{c,t}$

Minimizing peak: minimize m

where $m \geq \sum_c E_c x_{c,t}$

Contents

Minimizing peak when $E_c = 1$

Decision problem

$$A_{c,t} \le \sum_{i \le t} x_{c,i} \le B_{c,t}$$

$$\sum_{c} x_{c,t} \le M \text{ for all } t \in \{1, \dots, T\}$$

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Dror, Kubiak, Dell'Olmo

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Job scheduling

- WLOG: $A_{c,T} = B_{c,T}$

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Job scheduling

- WLOG: $A_{c,T} = B_{c,T}$
- Job is a pair (c, j) where $j \in \{1, \ldots, A_{c,T}\}$

Minimizing peak when $E_c = 1$

Decision problem

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Problem $P_m | r_i, p_i = 1, \text{chains} | L_{\max}$ is solvable in polynomial time.

Job scheduling

- WLOG: $A_{c,T} = B_{c,T}$
- Job is a pair (c, j) where $j \in \{1, \dots, A_{c,T}\}$
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Minimizing peak when $E_c = 1$

Decision problem

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- Deadline for job (c, j) is minimal t such that $A_{c,j} = t$

Contents

Greedy algorithm for minimizing cost

Reformulated problem for single house

Given $A_1, \dots, A_T$ and $B_1, \dots, B_T$ satisfying $A_t \leq A_{t+1} \leq A_t + 1$ and $B_t \leq B_{t+1} \leq B_t + 1$ and $A_1, B_1 \in \{0, 1\}$, find $x_t \in \{0, 1\}$ for $t \in \{1, \dots, T\}$

$$\text{minimizing } \sum_t P_t x_t$$

such that

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Lemma (Feasibility)

There exists a feasible solution if and only if $A_t \leq B_t$ for every t.

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Lemma (Feasibility)

There exists a feasible solution if and only if $A_t \le B_t$ for every t.

Lemma (Existence of a solution with $x_{t^\star} = 1$)

The problem has a feasible solution satisfying $x_{t^\star} = 1$ for a given $t^\star$ if and only if $A_{t^\star - 1} < B_{t^\star}$.

Greedy algorithm for minimizing cost

Greedy algorithm when prices are non-negative

initialization: $x_t := 0$ for all t

for $t^\star$ *sorted by prices* **do**

  if $A_{t^\star-1} < B_{t^\star}$ **then**

    $x_{t^\star} := 1$

    Update bounds A_t and B_t

Greedy algorithm for minimizing cost

Greedy algorithm for general prices

initialization: $x_t := 0$ for all t

for $t^\star$ *sorted by prices* **do**

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Update

$$t_A = 1 + \max\{t; A_t = A_{t^\star-1}\}$$

$$A_t^\star = \begin{cases} A_t & \text{if } t < t_A \\ A_t - 1 & \text{if } t \geq t_A \end{cases}$$

$$t_B = \min\{t; B_t = B_{t^\star}\}$$

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Greedy algorithm for minimizing cost

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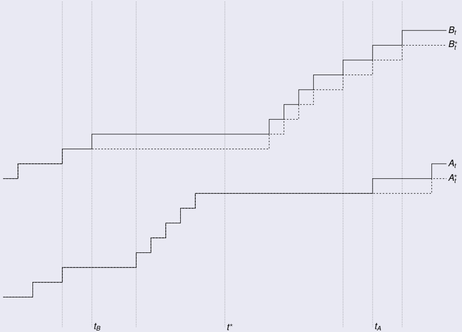

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Greedy algorithm

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Complexity

- Direct implementation: $\mathcal{O}(T^2)$
- Using binary trees: $\mathcal{O}(T \log T)$
- Using union-find data structure: $\mathcal{O}(T\alpha(T))$ + sorting

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Approximation

We find a solution $x^{A}_{c,t}$ such that

$$\max_t \sum_c E_c x^{A}_{c,t} \leq E + \max_t \sum_c E_c x^{R}_{c,t}$$

where $E = \max_c E_c$.

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Other objective functions

Minimizing peak (basic version)

Minimize m where $m \geq \sum_c E_c x_{c,t}$.

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Minimize $m_u - m_l$ where $m_l \leq F_t + \sum_c E_c x_{c,t} \leq m_u$.

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Minimize $m_u - m_l$ where $m_l \leq F_t + \sum_c E_c x_{c,t} \leq m_u$.

The approximation error for minimizing the fluctuation

$-E + \min_t F_t + \sum_c E_c x^{\star}_{c,t} \leq \min_t F_t \sum_c E_c x^A_{c,t}$ and
$\max_t F_t + \sum_c E_c x^A_{c,t} \leq E + \max_t F_t \sum_c E_c x^{\star}_{c,t}$

Algorithm

Basic steps

1. Preprocessing
2. Solve relaxed problem
3. Round all non-integer values in the optimal relaxed solution

Properties of the relaxed solution

Polytope P

(A1) $A_{c,t} \leq \sum_{i \leq t} x_{c,i} \leq B_{c,t}$

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(C5) $x^{\alpha}_{c,t} = x_{c,t}$ otherwise

Bipartite graph G of non-integer values of x

Vertices

- Set of time intervals.

Bipartite graph G of non-integer values of x

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There is a sequence $(c_1, W_1), \dots, (c_k, W_k)$ of all vertices of the second partity of G such that for every i vertex (c_i, W_i) has at most one non-leaf neighbour t_i in the graph

$$G_i = G \setminus \{(c_{i+1}, W_{i+1}), \dots, (c_k, W_k)\}.$$

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Improve approximation error

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- converters with three states: heating water for domestic hot water demands, space heating or off,
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- other electrical device you (may) have at home?