On nominal automata and their languages to verify interactive computation

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(joint work with Alexander Kurz and Emilio Tuosto)

Automata and formal languages for computational behaviours

Review: (classical) finite automata

Alphabet Σ: a finite set of letters, e.g. $\Sigma = \{a, b, c\}$
Automaton $\mathcal{A}$: a tuple $\langle Q, q_0, \delta, F \rangle$

- Q: a finite set of states
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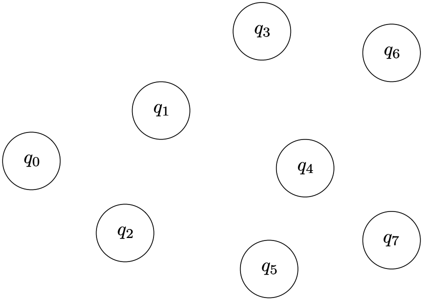

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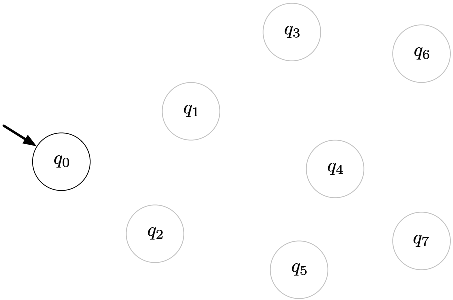

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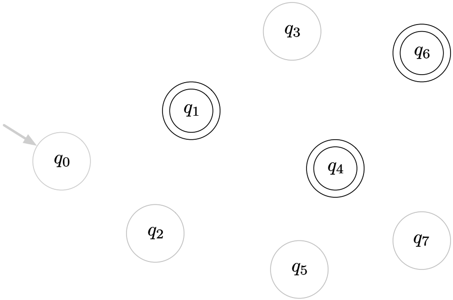

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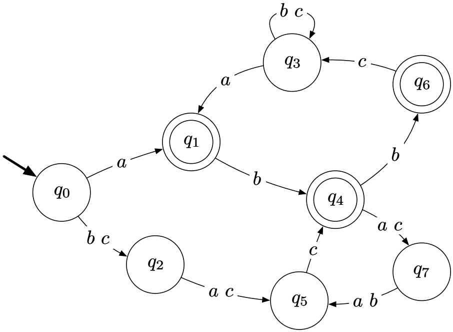
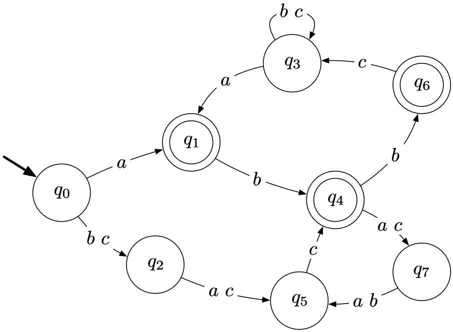


Review: (classical) finite automata

Word w: a finite sequence of letters in Σ, e.g. $w = abbca$
Language $\mathcal{L}$: a collection of words
For example, the following automaton $\mathcal{A}$



A word $a\ b\ c\ b\ c\ b\ c\ a$ is accepted? or rejected?

Automata: monitors for interactive computational behaviours

Monitoring for static computing

How to detect illegal behaviours on automata?

Example

Let a and b be possible actions and the following constraint:

Two consecutive actions should not be the same.

Safe behaviours: 'a', 'bab', '$ababab$', etc.
Bad behaviours: 'abb', '$baaab$', etc.

Monitoring for static computing

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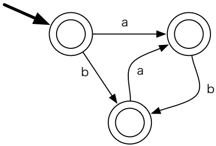

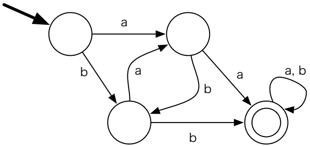

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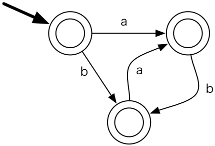

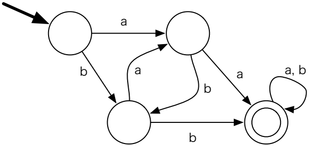

The monitor detects the malicious behaviours.

$$\underbrace{\cdots a\ b\ a\ b\ a\ b\ a\ b}_{\text{safe}}\ \underbrace{\boxed{a\ a}}_{\text{illegal}}\ \cdots$$

Research question

Is classical automata theory enough to monitor interactive computations?
Let's discuss this question by comparing with the R.Milner's argument

Environment-aware designs

$$\text{Communications: } \begin{cases} \text{not functional,} \\ \text{not prescribed,} \Longrightarrow \text{"environment-aware"} \\ \text{nor deterministic.} \end{cases}$$

Environment-aware designs:

- infinitely many components concurrently moving
- systems must react to the dynamic environments (all actions are not prescribed)

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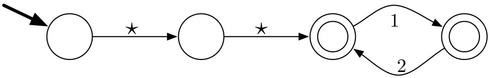

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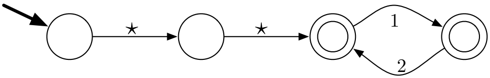

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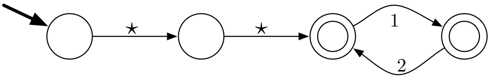

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Also, '*xyxy*' is described: '*x*' registered in 1 and '*y*' registered in 2.

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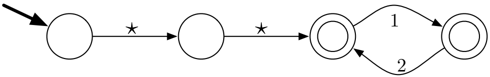

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Environment-aware designs provide schematic pattern matching.

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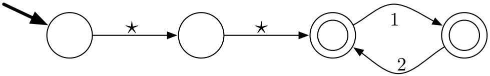

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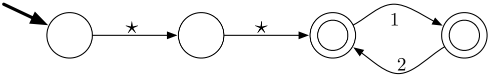

It stops at the first '*c*' in '*ababcbc*', although '*aba*' and '*bcbc*' are safe computations.

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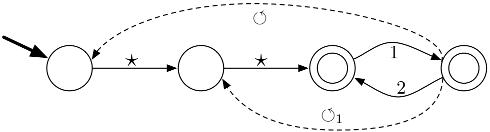

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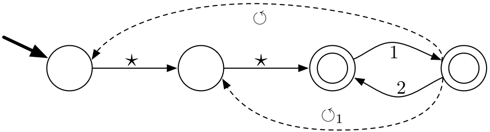

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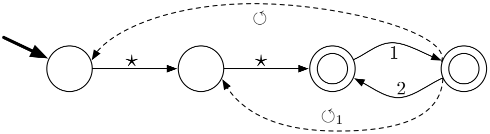

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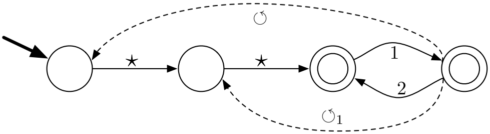

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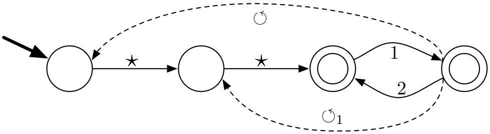

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- let the monitor "guess" $\Longrightarrow$ non-determinism and when the monitor detects ill-behaviours

Nominal automata

Related models

- N.Kaminski & M.Francez, “Finite-memory automata”
- U.Montanari & M.Pistore, “History-dependent automata”
- N.Tzevelekos, “Fresh-register automata”
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Key idea: automata with resources

$\Longrightarrow$ nominal computation theory

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Basic nominal automata $A^\sharp$

$\mathcal{H} = \langle Q, I, q_0, F, tr \rangle$

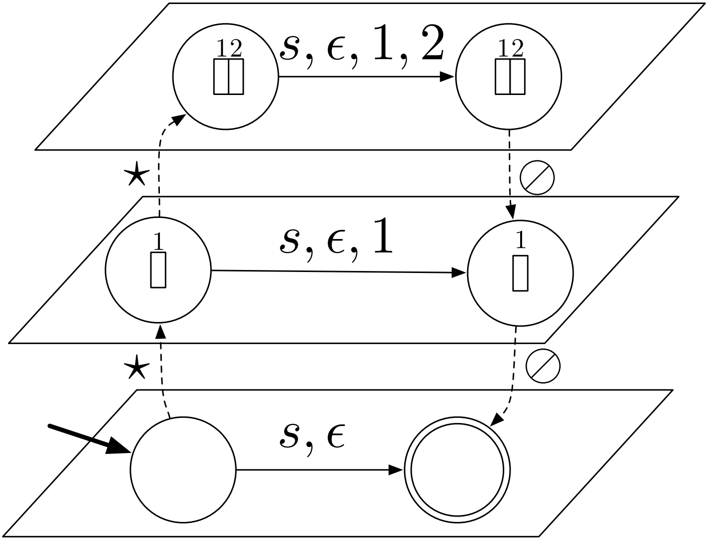
1. Q: (finite) named set (endowed with a function $\|\_\| : Q \to \mathbb{N}$) and we let $reg(q) := \{1, \dots, \|q\|\}$
2. I: input function

$$I(q) := \Sigma \cup reg(q) \cup \{\star, \oslash\}$$

3. q_0: initial state with no memory cell ($reg(q_0) = 0$)
4. F: final states with no memory cell ($reg(q) = 0$ for $q \in F$)
5. tr: transition relations satisfying for $q, q' \in Q$ and $\alpha \in I(q) \cup \{\epsilon\}$,

$$q' \in tr(q, \alpha) \iff \begin{cases} \|q'\| = \|q\| + 1 & \alpha = \star \\ \|q'\| + 1 = \|q\| & \alpha = \oslash \\ \|q'\| = \|q\| & \text{otherwise} \end{cases}$$

Picture of $A^\sharp$



Basic nominal automata

A little bit more preliminaries:

- Alphabet Σ: a finite set of letters (constants)
- Name $\mathcal{N}$: an "infinite" set of resource identifiers
- Transitions include "resource-allocation $\star$" and "resource-deallocation $\oslash$"

A run of nominal automata is a sequence of configuration:
Configulation $\langle q, w, list \rangle$:

- q: a state
- w: a word (sequence of letters etc)
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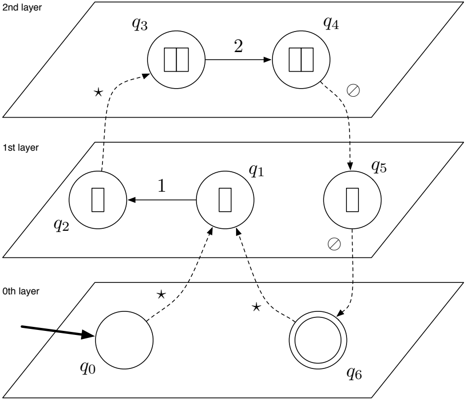

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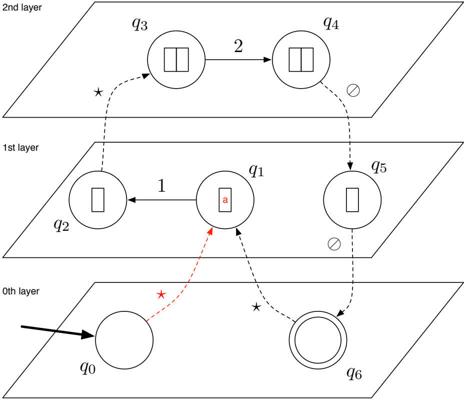

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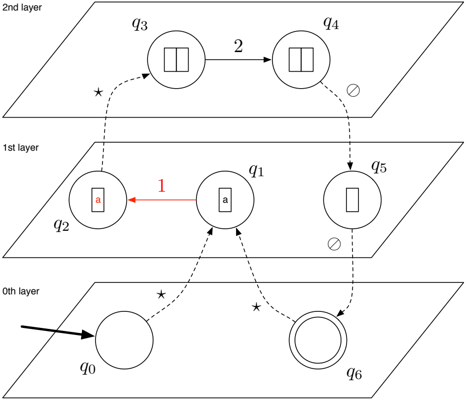

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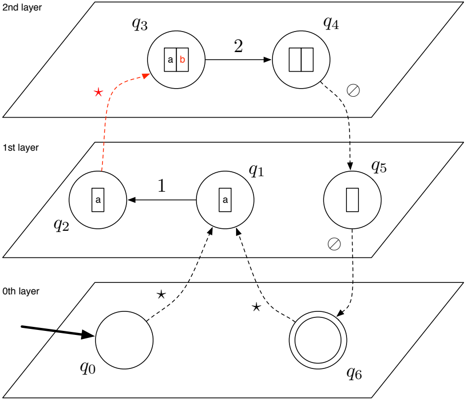

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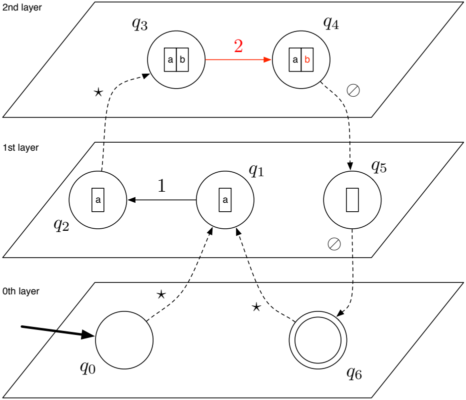

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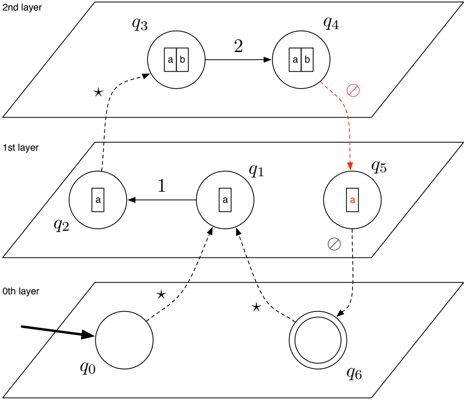

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$\xrightarrow{\oslash} \langle q_6, caaded, [] \rangle \xrightarrow{\star} \langle q_1, caaded, [c] \rangle$

$\xrightarrow{1} \langle q_2, aaded, [c] \rangle \xrightarrow{\star} \langle q_3, aaded, [c, a] \rangle$

$\xrightarrow{2} \langle q_4, aded, [c, a] \rangle \xrightarrow{\oslash} \langle q_5, aded, [c] \rangle$

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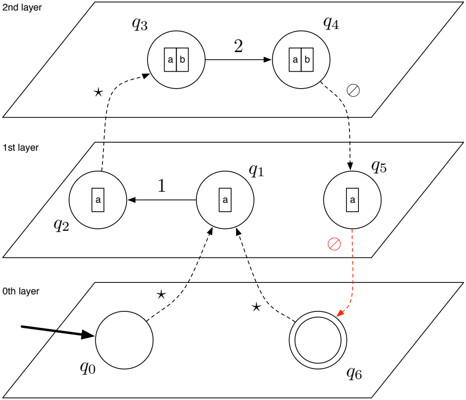
Example ($\Sigma = \emptyset$, $\{a, b, c, d, e\} \subseteq \mathcal{N}$)



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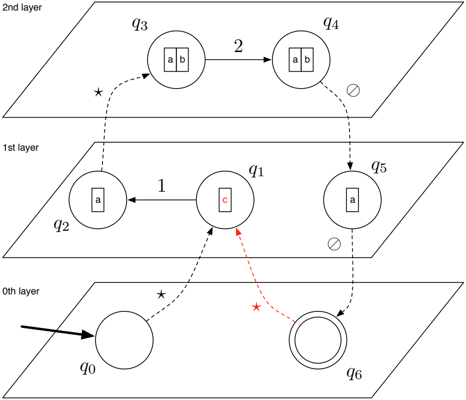

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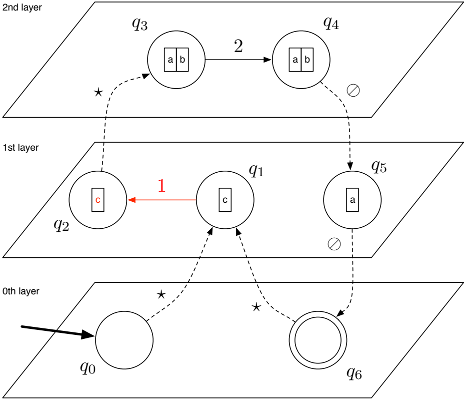

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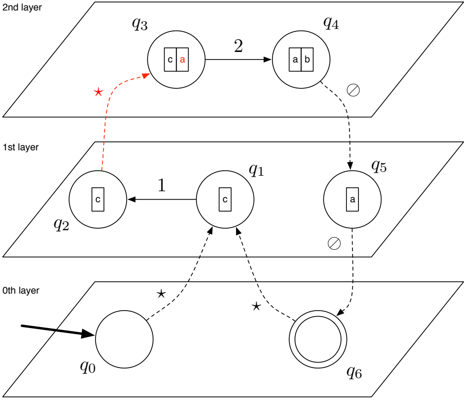

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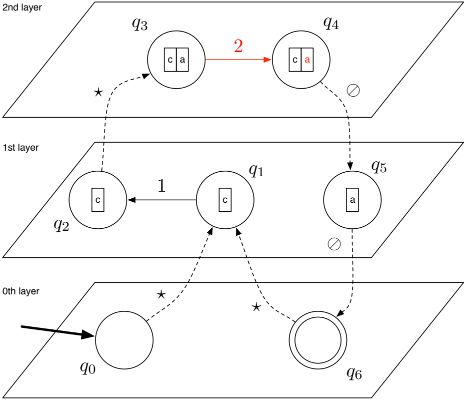

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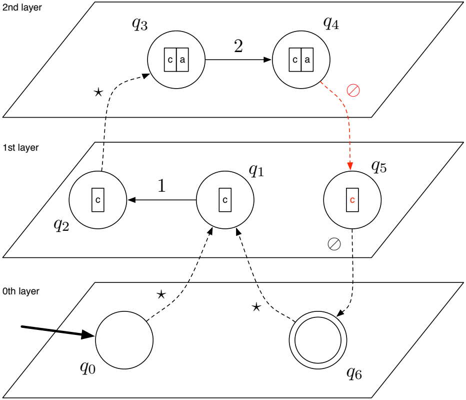

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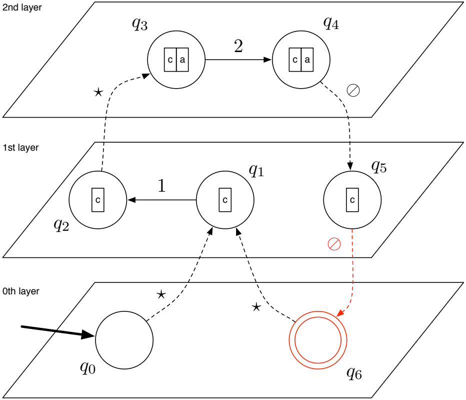

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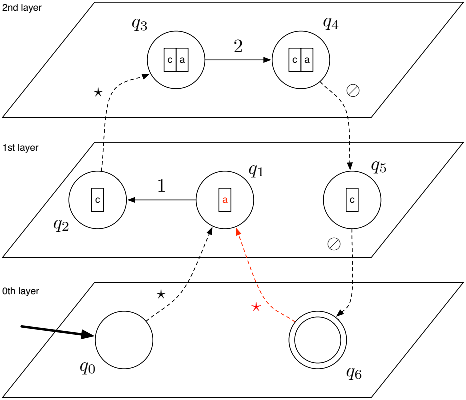

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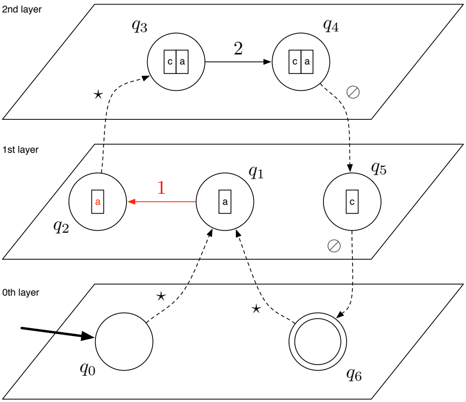
Our automaton for this example



$\xrightarrow{1} \langle q_2, ded, [a], \rangle \xrightarrow{\star} \langle q_3, ded, [a, d]\rangle$

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$\xrightarrow{\emptyset} \langle q_6, ed, []\rangle \xrightarrow{\star} \langle q_1, ed, [e]\rangle$

$\xrightarrow{1} \langle q_2, d, [e]\rangle \xrightarrow{\star} \langle q_3, d, [e, d]\rangle$

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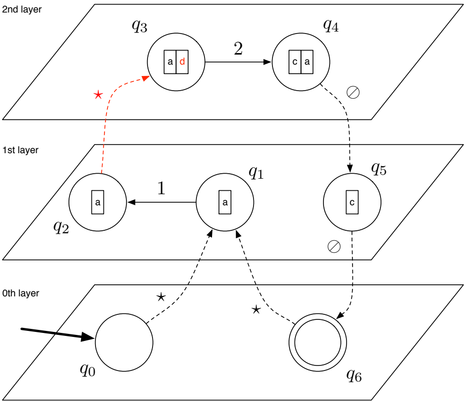

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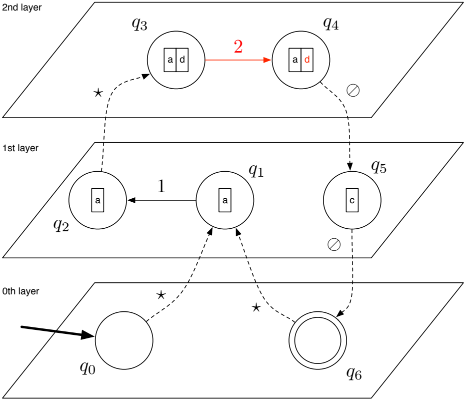

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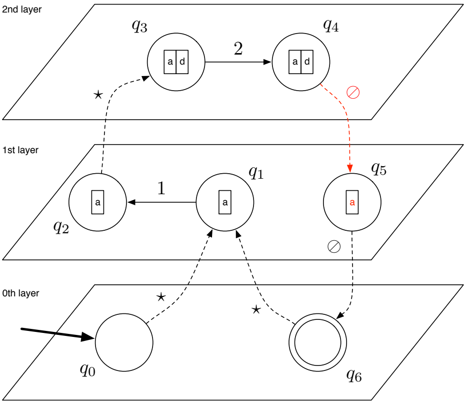

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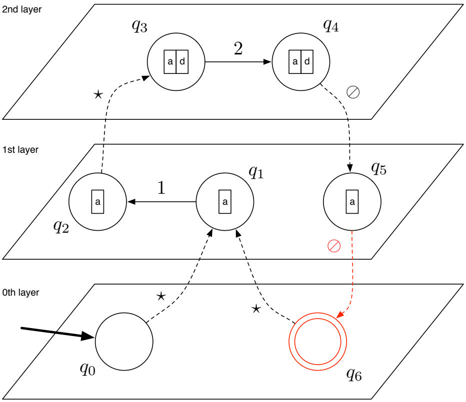

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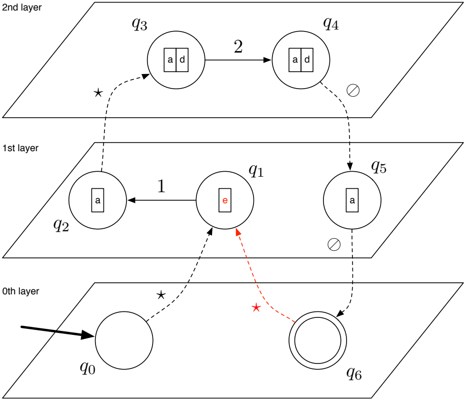

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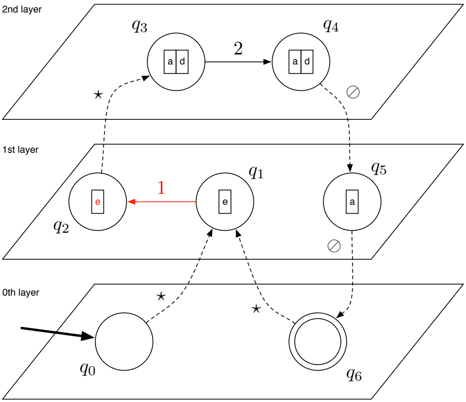

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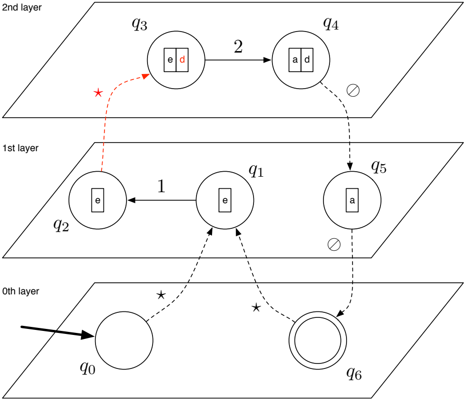

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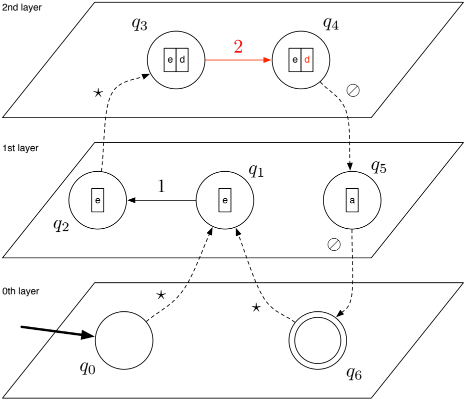

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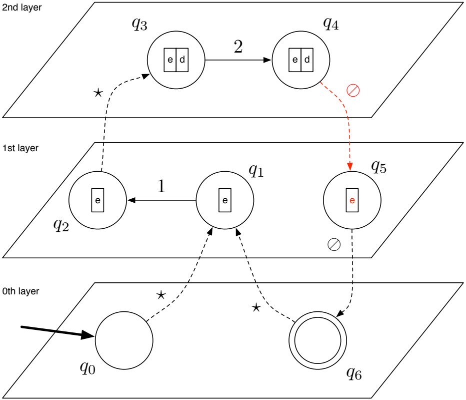

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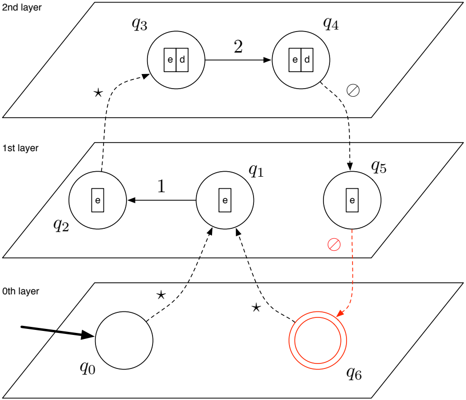

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A simple example (cont.)

Let $\mathcal{N}$ be an infinite set of letters: $a, b, c, d, e, \ldots \in \mathcal{N}$.

Many words exhibit the same pattern: for example,

$$\underbrace{ab}_{\neq}\underbrace{ab}_{\neq}\underbrace{ab}_{\neq}\underbrace{ab}_{\neq} \qquad \underbrace{ab}_{\neq}\underbrace{cd}_{\neq}\underbrace{ef}_{\neq}\underbrace{gh}_{\neq} \qquad \underbrace{ba}_{\neq}\underbrace{ca}_{\neq}\underbrace{ab}_{\neq}\underbrace{ac}_{\neq}$$

$$\underbrace{ab}_{\neq} \quad \underbrace{ba}_{\neq} \quad \underbrace{cd}_{\neq} \quad \underbrace{ab}_{\neq}\underbrace{ab}_{\neq} \quad \underbrace{ab}_{\neq}\underbrace{ab}_{\neq}\underbrace{ab}_{\neq}\underbrace{ab}_{\neq}\underbrace{ab}_{\neq}$$

But the following words do NOT: aa $\quad abaade$ $\quad abc$

Languages over infinite alphabets

$$\bigcup_{k \in \mathbb{N} \setminus \{0\}} \{n_1 \cdots n_{2k} \in \mathcal{N}^* \mid \forall 0 < i \leq k.\, n_{2i-1} \neq n_{2i}\}$$

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But the following words do NOT: aa $\quad ab\,aa\,de$ $\quad ab\,c$

Languages over infinite alphabets

$$\bigcup_{k \in \mathbb{N} \setminus \{0\}} \{ n_1 \cdots n_{2k} \in \mathcal{N}^* \mid \forall 0 < i \leq k.\, n_{2i-1} \neq n_{2i} \}$$

Languages on nominal automata

What are "the" words (and languages)?

There are a couple of different notions of words:

- sequences of letters and names
- words with explicit binders (resource allocation and deallocation)
- orbits
- schematic words

Different notions of words enjoy different mathematical properties:

- determinism
- (regular) expressions
- relationships between languages
- closure properties
- etc

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Our research directions, results and open questions

Nominal regular expressions (b-NREs)

$$ne ::= 1 \mid 0 \mid s \in \Sigma \mid n \in \mathcal{N} \mid ne + ne \mid ne \circ ne \mid ne^* \mid \langle_n ne \rangle_n$$

Example

$$\dfrac{\dfrac{\dfrac{[a] \vdash a \qquad \dfrac{[a,b] \vdash b}{[a] \vdash \langle_m m \rangle_m}}{[a] \vdash a \circ \langle_m m \rangle_m}}{[] \vdash \langle_n n \circ \langle_m m \rangle_m \rangle_n} \qquad \dfrac{\dfrac{[c] \vdash c \qquad \dfrac{[c,d] \vdash d}{[c] \vdash \langle_m m \rangle_m}}{[c] \vdash c \circ \langle_m m \rangle_m}}{[] \vdash \langle_n n \circ \langle_m m \rangle_m \rangle_n}}{[] \vdash \langle_n n \circ \langle_m m \rangle_m \rangle_n \circ \langle_n n \circ \langle_m m \rangle_m \rangle_n}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{[a] \vdash a}{[a] \vdash (\!| a \mid |\!)} \qquad \dfrac{\dfrac{[a,b] \vdash b}{[a,b] \vdash (\!| b \mid |\!)}}{[a] \vdash (\!| \star_1 \mid \star_1 \neq a |\!)}}{[a] \vdash (\!| a\star_1 \mid \star_1 \neq a |\!)}}{[] \vdash (\!| \star_2\star_1 \mid \star_1 \neq \star_2 |\!)} \qquad \dfrac{\dfrac{\dfrac{[c] \vdash c}{[c] \vdash (\!| c \mid |\!)} \qquad \dfrac{\dfrac{[c,d] \vdash d}{[c,d] \vdash (\!| d \mid |\!)}}{[c] \vdash (\!| \star_3 \mid \star_3 \neq c |\!)}}{[c] \vdash (\!| c\star_3 \mid \star_3 \neq c |\!)}}{[] \vdash (\!| \star_4\star_3 \mid \star_3 \neq \star_4 |\!)}}{[] \vdash (\!| \star_2\star_1\star_4\star_3 \mid \star_1 \neq \star_2, \star_3 \neq \star_4 |\!)}$$

The language is $\left\{ n_2 n_1 n_4 n_3 \in \mathcal{N}^4 \mid n_1 \neq n_2, n_3 \neq n_4 \right\}$.

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$$[a,b] \vdash b \qquad [c,d] \vdash d$$

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Extensions of basic nominal automata $A^\sharp$

Nominal automata with flexible deallocations $DA^\sharp$

- the order of deallocations is flexible
- a typical language

$$\mathcal{L}_{two} := \bigcup_{k \in \mathbb{N}} \{n_0 n_1 \cdots n_k \mid \forall i < k.\ n_i \neq n_{i+1}\}$$

Nominal automata with chronicles $CA^\sharp$

- the history of resources is kept
- a typical language

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Nominal automata with flexible deallocations and chronicles

- $DA^\sharp$ and $CA^\sharp$

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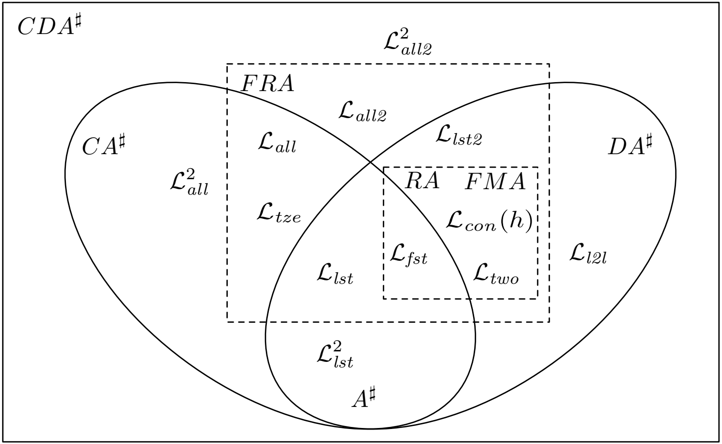
Description of languages over infinite alphabets on different nominal automata



Automata-Language game

Idea: Proponent $\mathbb{P}$ provides automaton $\mathcal{A}$ and Opponent $\mathbb{O}$ gives a counterexample

1. $\mathbb{P}$ chooses an automaton $\mathcal{A}$ in $\mathcal{C}$
2. $\mathbb{O}$ chooses a word w in $\mathcal{L}$
3. $\mathbb{P}$ exhibits a path to accept w or revise $\mathcal{A}$ to $\mathcal{A}' \in \mathcal{C}$
4. $\mathbb{O}$ adds a suffix v so that $wv \in \mathcal{L}$
5. repeat from Step 3

If $\mathbb{O}$ has a winning strategy, $\mathcal{L}$ cannot be accepted by any automaton $\mathbb{A}$ in the class of automata $\mathcal{C}$.

Theorem

- *$\mathcal{L}_{all}$ is not accepted by $DA^\sharp$*
- *$\mathcal{L}_{two}$ is not accepted by $CA^\sharp$*

Open problems and further reserach directions

Technical open problems

- Presentations and expressions of words and languages
- General separation method (partially solved: language-languae game)
- Communicating models and frameworks
- Nominal grammer and effective algorithm

Further research directions

- Enrichments on resource structures: e.g. not just $=$ and $\neq$ but also with security levels or time-stamps
- Safety properties over mobile interactions: how to inductively guarantee safety properties over mobile interactions
- Schematic pattern matching on large data: schematic pattern matching to calculate similarities

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